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A PRACTICAL METHOD FOR DEMONSTRATING THE ERROR OF MEAN SQUARE.

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The standard deviation, or error of mean square, employed in biological and in other measurements, as the most approved criterion or measure of the probable error—or, when taken with respect to biological data—of variation, so-called, in the gross, has a meaning which is difficult of explanation to non-mathematical students. The writer has found it advantageous, in teaching classes in plant breeding, to make a certain amount of use of biometric studies, before taking up work in genetics. In this connection, it has been found desirable to make such an explanation of such constants as the standard deviation and the coefficient of correlation, as would answer for scientific purposes, in the case of agricultural students who have not gone far in mathematics.

THE QUESTION OF ERROR.

In all physical and biological measurements and observations, there enters in the factor of experimental error, due to the environment, the apparatus, or the observer. In the latter case, the errors may be due to fatigue, cold, nervousness or other temporary disability, or to the constitutional bias known as the personal equation. Some errors, of course, are simple mistakes, as in reading off the wrong figure—mistaking a 3 for a 5, etc. Leaving out the constant errors—"errors which, in all measures of the same quantity, made with the same care, and under the same conditions, have the same magnitude, or whose presence and magnitude are due to some fixed cause," (1 p. 2), we have the category of so-called accidental errors with which the standard deviation has to deal. The constant errors, include those due to the physical environment, e. g. the expansion and contraction of the glass of a mercury thermometer under different

temperatures; those due to defects in the instrument used, as in the behavior of an analytical balance, of a micrometer screw, etc., and finally those due to the personal equation. Each of these classes of error can be definitely ascertained (within a certain degree of probable error again), and allowed for in the computations.

Every measurement, however apparently absolute it may be, is a relative thing, made in terms of something else that is also fallible—that is to say, that is subject to variation. All scientific measurements, in other words, are made in terms of units theoretically invariable, but which are however, always practically applied by means of a mass of matter used to measure with, and which, itself, of necessity, is a more or less variable quantity—how variable, being again, in turn, a matter of determination.

Having eliminated, or allowed for the constant errors, it is necessary to consider those due to accidental causes—"whose effect upon the observations is not determined by any circumstances peculiar to that particular set of measurements, and which cannot therefore be computed and allowed for beforehand. Such errors are those due to sudden changes in defraction owing to sudden and unobserved changes in temperature; unequal expansion of different parts of an instrument with change in temperature, shaking of an instrument in the wind, etc. But most important of all, are those errors which arise from imperfections in the sight, hearing, and other senses of the observer, which render it impossible for him to adjust and use his instruments with absolute accuracy. After a full investigation of the constant errors, . . . the problem now remains to combine the observations so that the remaining accidental errors shall have the least probable effect upon the results, and it is to bring about this combination of observations, that we employ the Method of Least Squares." (1, p. 3.)

As Bartlett further says, "All constant errors are supposed to have been eliminated before the Method of Least Squares is applied in deducing the results." (1, 89.)

Merriman expresses similar views. "Accidental errors are those that still remain, after all constant errors, and all evident mistakes have been carefully investigated, and eliminated from the numerical results." . . . These are the errors that appear in all numerical observations, however carefully the measurements be made, and whose elimination is the object of the Method of Least Squares." (8, p. 4.)

However, the elimination of constant causes of error is an exceedingly difficult matter, and, as Holman says, (5) "often proves to be a surprisingly large amount, in spite of most painstaking efforts for its elimination. For any specific method or apparatus will have its own characteristic set of errors, of which some will be predominant, and will determine a constant error, more or less large in the results obtained. Another method, will, on the other hand, be characterized by a different set of sources of error, and will have a different constant error. Observations taken under diverse conditions with the same method, will also have differing constant errors, and finally, different observers will show different personal equations. Thus, results obtained by changing method, apparatus, observers, or other conditions, will materially differ in the sources and amounts of their constant errors. The greater the number, and the more complete the variety of the changes, the greater becomes the diversity of the sources of error." (pp. 7-8.)

THE ARITHMETICAL MEAN.

When sources of error exist, and the true value is not known, the best possible measure of that value, is the arithmetical mean. So far as the so-called accidental errors are concerned, it is found that they fall around an ascertained center; that positive errors are about as frequent as negative ones of the same magnitude; that large errors are less frequent than small ones; and that very large errors seldom occur. The center of balance about which the errors or observation fall, is known as the arithmetical mean.

"The use of the arithmetical mean, as the best representative value, (in a large series of observations taken with equal care under the same conditions) can be considered as in accord both with the theory of probabilities, and with practical experience. But its employment is, however, justifiable, not in large series only, but in small ones as well. For, although the reliability or degree of probability of the mean in a small series will be less than in a larger one, yet the mean has a greater probability even in a very small series, than any other representative value which can be indicated." (5, p. 16.)

The attempt to obtain the true value or magnitude of a scientific measurement, has been well compared by Clark Maxwell to shots at a target—the misses constituting the errors. For illustration of the principle of the mean, we may take Sir William Crookes' investigation of the atomic weight of thallium (3).

The results of the different determinations, arranging them in the order of their increasing magnitudes, were

203.628	203.632	203.636	203.638	203.639
203.642	203.644	203.649	203.650	203.666

Out of the ten discrepant results, it is impossible to ascertain the true value. What is the best representative of that value? Experience has shown the arithmetical mean to be the best—i. e., the most representative value of a series of observations made under the same conditions—all being equally reliable. The arithmetical mean is so regarded, because it is a value, the deviations from which, in the plus and minus directions, being equally probable, will cancel one another. Let X_1, X_2, X_3 . . . X_n be a series of observations, A being their mean. Then the algebraic sum of the differences of all these observations from the mean, will equal zero. Or, $(X_1 - A)$ $(X_2 - A)$ $(X_3 - A)$ . . . $(X_n - A) = O$. Or, to quote Bartlett's expression (5, p. 4), "In a series of direct observations, M, M_2 . . . M_n, is made upon the value of the quantity M, all the observations being made with the same care, and under the same circumstances, the most probable value M_o, of that quantity, is the arithmetical mean of the observations, or,

$$\frac{M_o = M_1\,M_2\ \ .\ \ .\ \ .\ \ M_n}{n} = \frac{M_n}{n}$$

In general, *the mean is that number which makes the differences between itself and the measurements made in its determination the smallest possible.*

To quote from various writers on the characteristics of the arithmetical mean:

"The most probable value of a quantity which is observed directly several times with equal care, is the arithmetical mean of the measurements. The average, or arithmetical mean, has always been accepted and used as the best rule for combining direct observations of equal precision, upon one and the same quantity. . . . If the measurements be but two in number, the arithmetical mean is undoubtedly the most probable value; and, for a greater number, mankind, from the remotest antiquity, has been accustomed to regard it as such.

"It is a characteristic of the mean, that it renders the algebraic sum of the residual errors zero." (8, p. 22.)

"As residual is the difference between the most probable value of the observed quantity, and the measurement upon it. This most probable value is that deduced by the application

of the Method of Least Squares to the observations; for instance, in the simple case of direct measurements on a single quantity, the arithmetical mean is the most probable value." (8, p. 5.)

Or, as Bartlett puts it:

"The residual of an observation is the difference between the observed value of the measured quantity, and the value rendered most probable by the existence of the observations." (1, p. 5.)

According to Legendre, "The most probable value for the observed quantities is that for which the sum of the squares of the individual errors is a minimum." (7, p. 509.)

This value will be a maximum, when the sum of the squares of the residuals is a minimum.

"The most probable values of unknown quantities, connected by observation equations, are those which will render the sum of the squares of the residuals arising from the observation equations, a minimum." (7, p. 71.)

"Hence, the adjustment of observations by the Method of Least Squares is based upon the principle that the most probable system of values of the unknowns *is that which renders* the sum of the squares *of the residuals a minimum.*" (1, p. 15.) (Italics inserted.)

"The most probable value of a measured quantity that can be deduced from a series of observations made with equal care and skill, is that for which the sum of the squares of the residuals is a minimum." . . . "This is the so-called most probable value, and it is the office of the principle of Least Squares, in any case, to point out the way of arriving at it." (7, p. 59.)

"The most probable value of the observed quantity, a, in the case of observations supposed equally good is that which *assigns the least possible value to the sum of the squares of the residual errors.*" (6, p. 28.) This is the statement in its simplest form, of the principles of Least Squares. The rule of the arithmetical mean follows directly from the principle of Least Squares."

"The most probable value of the observed quantity, or simply the probable value of the observed quantity, or simply the probable value, in the ordinary sense of the expression, signifies that which, in our actual state of knowledge, we are justified in considering as more likely than any other, to be the true value. In this sense, the arithmetical mean is the most probable value which can be derived from observations considered equally good. This is, in fact, equivalent to saying that we accept the arithmetical mean as the best rule for combining the observations,

having no reason either theoretical or practical, for preferring any other." (6, p. 6.)

"That the most probable value, when there are but two observations, is their arithmetical mean, follows rigorously from the hypothesis, that positive and negative errors are equally probble." (Quoted by Johnson, 6, p. 7.)

"The universal practice of taking the arithmetical mean of all measures of a single quantity, as the best value of that quantity, is a particular case under the more general Method of Least Squares." (2, p. 16.)

Comstock gives (2, pp. 15-16) an interesting mathematical discussion of the reason why the arithmetical mean is the best possible measurement of a quantity whose true value is unknown but which is approached through a series of observations.

Finally as Holman says (5, p. 16), "In a large series of equally careful observations of the same quantity, under the same conditions, the variable parts of the errors will be sensibly eliminated, by arranging the results, that is, by the employment of the mean as a representative value. The law of deviations already stated shows that to be true, and as this law has been arrived at by an application of the theory of probabilities, and confirmed by the results of specific as well as of general experience, the use of the arithmetical mean as the best representative value in such a large series, can be considered as in accord both with the theory of probabilities, and with practical experience. But its employment is, however, justifiable, not in the large series only, but in small ones as well. For although the reliability or degree of probability of the mean in a small series will be less than in a larger one, yet the mean has a greater probability, even in a very small series, than any other representative value which can be indicated." (p. 16.)

The reliability of the mean is considered to be in proportion to the square root of the number of observations in the series, but Holman (5, p. 16), cautions against "attaching undue weight to this numerical relation, when the number of observations is very small, as for instance, when not exceeding five or ten. A similar caution should be urged, respecting all applications of the method and rules of Least Squares, when it is small, although the use of the methods in such cases is fully justified by the fact that they give the best results obtainable."

A mathematical discussion of the mean is given in Weld (10, pp. 58-60.)

Considerable space has been given up to an exposition of the significance of the arithmetical mean, since it is the fundamental basis upon which rests the concept of error in a series of observations—in other words, the "mean," and the "error of mean square," or "standard deviation," are mutually interdependent factors.

Referring now to Crookes' investigation before-mentioned, we find the mean of the ten determinations to be 203.642. This value is not the true value of the atomic weight or thallium, but is simply one of an infinite number of possible determinations, which might, for experimental purposes, be assumed to lie between the upper and lower limits, 203.666 = 203.628, with a likelihood of closely approximating the arithmetical mean—203.642—demonstrated by the size of the probable error.

As Mellor says (7), "It is practically useless to define an error as the deviation of any measurement from the true result, because that definition would imply a knowledge *which is the object of investigation*"(italics inserted). "What then is an error? Before we can answer this question, we must determine the most probable value of the quantity measured. The only available data, as we have just seen, are always associated with the inevitable errors of observations. The measurements, in consequence, all disagree among themselves within certain limits. In spite of this fact, the investigator is called upon to state definitely what he considers to be the most probable value of the magnitude under consideration. Indeed, every chemical or physical constant in our text books is the best representative value of a more or less extended series of discordant observations." (7, p. 510.)

Mellor cites an interesting instance, which may be compared with the Crookes' case. "Giant attempts," as he says, have been made to determine the exact length of a column of pure mercury, having an area in cross-section, of one square millimeter, and a resistance of one ohm, at 0°C., (7, p. 510). The following data have been obtained:

Centimeters
106.19
106.21
106.24
106.27
106.29
106.31
106.32
106.32
106.33

The true value appears to be one of an infinite number of possible values, lying between 106.19 = 106.33, with the greatest probability of its lying very near the arithmetical mean of 106.28, how near, of course, is measured by the probable error of the mean.

Now, in the case of biological measurements or observations, such as the lengths of leaves on a tree, the number of ray-flowers on a sunflower head, the number of seed-pods on an alfalfa plant, or the number of rows on an ear of corn, there is no real true value, toward which the mean is an approximation. In the cases just mentioned, and others of a similar nature, the mean simply represents a center of equilibrium, or center of gravity, as it were, of the variations in the given organ or character, for a given time and locality. Having found that center of equilibrium—the arithmetical mean—the next problem is to ascertain what amount of swing or oscillation there may be on either side of this center. These are the variations, which correspond to the errors we have referred to, in the making of physical measurements. In the latter case, they are actual errors. In other words, by reason of the limitations of apparatus, of visual perception, or of other personal factors inherent in the investigator, or by reason of uncontrollable factors outside of either operator or instrument, there occur certain fluctuations in the measurements, which are errors, in the sense that all of them are deviations from the actual true value required. There is no true value existing, and forever approachable; and yet experimentally speaking, forever unattainable, with respect to the number of leaves on a tree, or the number of rows of kernels on an ear of corn. For every species, race, variety, or individual plant, for example, there is a certain center of equipoise with respect to leaf length. With every well-established strain of corn, for example, there is a center of equilibrium, with respect to the number of rows of kernels on the ear. The observed differences from any such center of equilibrium are called variations or errors, which seem to accord with that law of probability which expresses the manner in which the errors are found to occur in the case of physical and chemical measurements and determinations.

We come now to the methods used in the measurement of these biological deviations: in certain cases in biology, we may treat what we designate as variability in the same manner as, in the physical and chemical sciences, we treat what is there known as error.

The nature of the mean as a factor in biological measurements is well described as follows: (8, p. 173.)

"Quetelet and others have clearly established that stature and the other proportions of the body are governed by the law of the probability of error. Nature, in fact, aims to produce certain mean proportions, and the various groups into which mankind may be classified deviate from the mean according to the law of the probability curve. . . . The average man, says Quetelet, is for a nation what the center of gravity is for a body; to the consideration of this are referred all the phenomena of equilibrium."

Variability, in a biological sense, depends upon a number of causes, the most important of which are genetic in nature, i. e., depend upon the combinations existing in the germ cells. In plant breeding, the main problem is to find out what these various combinations are; and in work of this kind, it is seldom necessary to make use of the standard deviation.

Nevertheless, not only in gauging the general evolutionary, trend of a group, but also in practical work in plant selection, not involving crossing, or the analysis of genetic factors—such work for example as farmers and agronomists are doing all of the time, in effecting the practical improvement of a race or strain of agricultural plants—the use of biometric measurements is eminently useful, and the writer believes that, for this reason, work of this sort still has a proper place in courses in plant breeding in agricultural schools.

In explaining the significance of the chief biometric constant the standard deviation, or error of mean square, the mathematical demonstration of its character need not be entered into for the purpose of the average student. Briefly speaking, it is the square root of the average of all the squared deviations from the mean. In other words, each error, or deviation from the mean, is squared, and multiplied by the number of times it occurs; the sum of these products is then taken, divided by the total number of cases, and the square root of the quotient obtained. The standard deviation, therefore, is the square root of the average of the squares of all the deviations from the mean. It is considered to be the best measure of variability. As Pearson says, "Other measures of deviation have been devised, some of which are occasionally useful, but for theoretical and practical reasons, the standard deviation may be considered the best. It is not hard to find, and it occurs and recurs in all sorts of

investigations. The following is the rule to obtain it. *Multiply the frequency with which each individual type occurs, by the square of its deviation from the mean; add all of the products together, and divide by the total number of individuals.* This is the square of the standard deviation." (9, p. 387.)

It is expressed by the formula,

$$\sigma = \frac{\sqrt{\Sigma (d^2 \cdot f)}}{n},$$

in which d is a deviation from the mean, f, the frequency of the occurrence, Σ, the sum of the series of products d x f, n the number of cases, and o the standard deviation.

An example of the computation of the error of mean square is given in the following table. The first column contains seventeen measurements of a certain wave length, made by Professor Rowland. The second column gives the residuals, or differences between each measurement and the mean (A = 4.5048). The third column gives the squares of the residuals. The sum of squares is taken without regard to sign.

V.	D.	D².
4.524	0.0185	.00034225
4.500	0.0055	.00003025
4.515	0.0095	.00009025
4.508	0.0025	.00000625
4.513	0.0075	.00005625
4.511	0.0055	.00003025
4.497	0.0085	.00007225
4.507	0.0015	.00000225
4.501	0.0045	.00002025
4.502	0.0035	.00001225
4.485	0.0205	.00042025
4.519	0.0135	.00018225
4.517	0.0115	.00013225
4.504	0.0015	.00000225
4.493	0.0125	.00015625
4.492	0.0135	.00018225
4.504	0.0005	.00000025
A = 4.5048		.00173825

.00173825/17 = 0.00101 (the standard deviation).

The mathematical significance of the standard deviation is two-fold; its relation to what is known as the normal probability curve, or frequency curve, or curve of error. and its relation to the mean. As to the former, it may be described as bearing the same relation to this curve that the radius of the circle bears to the circle. It limits the spread of the curve, so that when σ is small, the curve will be narrow and crowded; when σ is large, the curve will be broad and much spread out. It will be

apparent at once that the greater the variability, the more the curve will be spread or flattened out; and the less variable the organism is with respect to any given character, the more narrow and limited in its spread will be the frequency curve describing it. It is constant for any curve, and measures the variability of the curve, or the steepness of its slope. "It is expressed geometrically, as the half parameter, or the abscissa of the point on the frequency curve, where the change of curvature (from concave to convex toward the center), occurs." (4, pp. 16 and 23.)

The standard deviation has the advantage of being a concrete number, and of the same nature as the individuals measured. This is to say, if the measurements are of length, and have been made in centimeters, the standard deviation is also in centimeters. If the measurements are of weight, and have been expressed in pounds, the standard deviation is in pounds also. For this same reason, the standard deviations taken in different classes of experiments are not able to be compared with one another, any more than we can compare centimeters to pounds. For the purpose of reducing to a common denominator, as it were, the data as to variation obtained in different units of measurement, a measure has been devised called the "coefficient of variability," (C. V.). It is the ratio of the standard deviation on the mean, multiplied by 100. Reduced to a formula,

$$CV = \frac{\sigma}{A} \times 100.$$

The demonstration which the writer has used to bring the nature of the standard deviation graphically before elementary students involves the comparison frequently used in discussing the law of error, or marksmen firing at a target.

It is stated (7, p. 509), that formerly, in the Belgian army, the scores at rifle practice were determined by adding together the distance of each man's shots from the center of the target, the marksman having the smallest sum becoming the winner. Suppose, however, we had, in a series of ten shots, the following records. Let A and B be two riflemen:

Shot No.	Distances of shots from center of target, in inches. A	B
1	2	4
2	8	5
3	3	6
4	7	4
5	9	3
6	4	5
7	2	5

8	5	6
9	9	7
10	1	5
	50	50

According to the sum of the distances of their respective shots from the center of the target, A and B form a tie. When, however, we apply the test of the error of mean square to their shooting, we find that A has a standard deviation of 2.10 inches, while B has the much lower standard deviation of 1.09 inches, and he is thus shown to be the more reliable shot, as demonstrated by his lower variability. Even a casual examination of their respective shots shows plainly that B is really, as a matter of common sense, the better shot of the two, because, as the saying is, he bunches his shots. The following additional example of the target will illustrate in some detail the application of the standard deviation, or error of mean square.

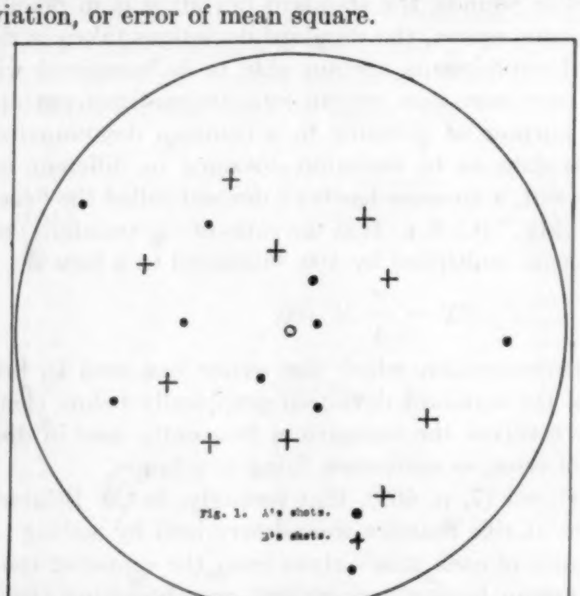


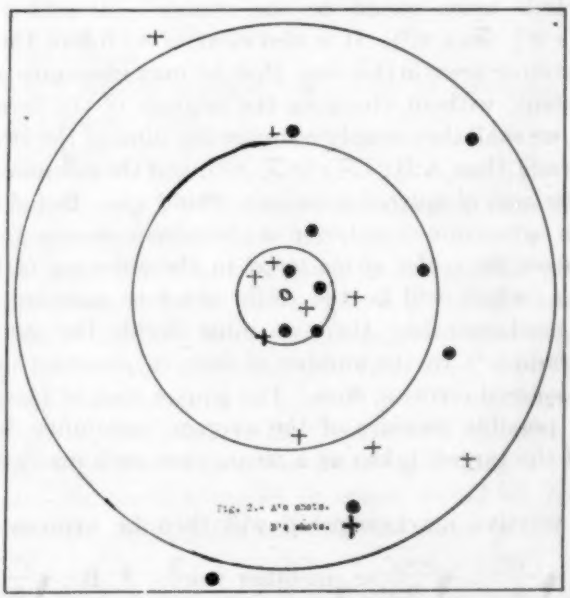
Fig. 1. A's shots. •
B's shots. +

The standard United States army target, used on rifle ranges at 200 to 300 yards distance, is an oblong, 4x6 feet, in the center of which are three concentric rings, of which the central one (called the bull's eye) has a diameter of 8 inches; the second a diameter of 26 inches; while the third or outermost ring has a diameter of 46 inches. The value in points, of shots placed in the several rings of the target, and of that portion of the target lying outside the rings, as established by the army regulations, is as follows:

First ring or bull's eye	5 points
Second ring	4 points
Third ring	3 points
Outside target	2 points

Each contestant is allowed ten shots. Let us suppose the following case, which may easily occur. Two riflemen, A and B, make records summing up to 39 points each, thus forming a tie, the individual shooting-records being as follows:

Shot No.	A's points	B's points
1	5	5
2	4	5
3	4	3
4	4	5
5	3	4
6	5	3
7	4	3
8	3	5
9	4	4
10	3	2
	39	39



Now, from an inspection of the above table, it is evident that while B has placed four shots in the bull's eye to A's two, nevertheless, A's record shows that his shots have been more uniformly placed near the center of the target than have B's, whose shooting, despite his four center shots, has been distributed promiscuously over the entire target. It is evident, as a matter of common sense, that A is the more reliable shot of the

two, and yet, according to the record, there is a tie, as in the case previously given.

Now, it is evident, as previously stated, that shots placed in a target may be taken in the sense of errors, or deviations from a mean (the mathematical center of the target being taken as the mean). The errors then are made in all directions from that center, and the distance of each shot from the center may be regarded in the sense of a deviation from the mean. If we regard the distance of a shot from the center of the target as a radius, then, since such a shot may be placed at any point in a circle around the center of the target, within the limits of its radius, we should treat the various misses or deviations from the center of the target as being proportionate to the area of a circle πr^2, of which r is the distance of the shot from the center. If we then determine the sums of circle areas formed by each man's shots taken as radii, it is plain that the man having the smaller total sum of circle areas would be the winner. In other words, $A:B::\Sigma(\pi \cdot r^2) : \Sigma(\pi \cdot r_1^2)$. It is also clear, if we follow the principle of least circle areas in this case, that we may eliminate π, which is a constant, without changing the relation of the terms. By so doing, we shall then simply compare the sums of the two sets of squared radii, thus, $A:B::(\Sigma r^2):\Sigma(r_1^2)$, and the rifleman, having the *smaller sum of squared deviations*, would win. But if we wish to obtain a measure or criterion of the *relative average fluctuation or miss from the center of the target* in the shooting of the two marksmen, which will be the really absolute measure of their relative marksmanship, then we must divide the sum of the squared radii (r^2), by the number of shots (n), in order to get the average *squared* error or miss. The square root of this will be the best possible measure of the average variability from the center of the target, taken as a mean, that each man's shooting displays.

Their relative marksmanship will then be expressed

$A:B::\sqrt{\frac{\pi r^2}{n}} : \sqrt{\frac{\pi \cdot r_1^2}{n}}$ or, in other words, $A:B::\sqrt{\frac{r^2}{n}} : \sqrt{\frac{r_1^2}{n}}$

The following example will serve to illustrate the method. Where r = *radius*, and f = the number of times each length of radius occurs, called the *frequency*.

No. of shots	Dist. of A's shots from math. center of target in inches, (r)	r^2	f	r^2f
1	3	9	1	9
2	7	49	1	49
3	5	25	1	25

4	6	36	1	36
5	14	196	1	196
6	2	4	1	4
7	10	100	1	100
8	15	225	1	225
9	8	64	1	64
10	24	576	1	576
				1284

$\Sigma r^2 \cdot f = 1284$
Av. $r^2 = 1284 \div 10 = 128.4$
$\sqrt{r^2} = 11.3$ inches

No. of shots	Dist. of B's shots, etc.	r^2	f	r^2f
1	3	9	1	9
2	4	16	1	16
3	15	225	1	225
4	2	4	1	4
5	6	36	1	36
6	21	441	1	441
7	14	196	1	196
8	3	9	1	9
9	12	144	1	144
10	25	625	1	625

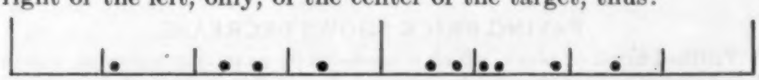
$\Sigma r^2f = 1705$
Av. $r^2 = 1705 \div 10 = 170.5$
$\sqrt{r^2} = 13.3$ inches

We thus have the relative variability in marksmanship expressed in concrete form.

A = 11.3 inches
B = 13.3 inches

A is thus shown to be the more certain shot than B, in that his shooting is, on the average, less variable than B's.

Now to bring this example closer to the general problem of variability in organisms, let us suppose that the marksmen aforesaid, instead of shooting at a round target, were shooting through a horizontal slit or opening, and that their rifles were fixed horizontally in a frame running on a track, in such manner that they could be moved for aiming, to the right or left, but not vertically. All of the shots would then be distributed along a horizontal target, and the various misses or errors would all be to the right or the left, only, of the center of the target, thus:



We now have a picture or diagram of individual variability as we find it expressed in organisms, in which the variations are to either side (plus or minus), of an assumed center called the mean, which is the center of our horizontal target. And here, as in the case of the target, the criterion, or measure of variability, is obtained by squaring all the distances from the center, or mean, i. e., all the errors, taking the average, and

extracting the square root; the formula, substituting d (deviation) for r (radius), being as previously given: $\sqrt{\frac{\Sigma d^2 \cdot f}{n}}$

It has seemed to the writer that the above illustration of the principle of the error of mean square is effective as a mode of presentation. So far as the writer knows, it has never been published.

It is by the graphic method described above that the writer has had the best success in giving to students a quasi-mathematical justification for the use of the error of mean square as the best measure of variation in a series of what is commonly called biological measurements.

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PAVING BRICK SHOWS DECREASE.

Vitrified brick or block, which is used chiefly as paving material, was in less demand in 1918 than for many years, the decline in its use being caused by high freight rates, embargoes on rail shipments of commodities least essential for war purposes, and shortage of fuel. The quantity marketed in 1918 was 403,512,000 brick or block, a decrease of 303,422,000, or forty-three per cent. The value was $7,232,000, a decrease of $3,433,000, or thirty-two per cent. The production in 1918 was the lowest since 1896 and was only about forty per cent of that of the maximum in 1909. The value of this product in 1918 was the smallest since 1905, though the average price per thousand ($17.92) was the highest recorded.

THE ADDITION OF PHYSICAL QUANTITIES—A SUGGESTION TO TEACHERS OF ELEMENTARY PHYSICS.

By Will C. Baker,

Queen's University, Kingston, Ont.

Note: *The following is offered in suggestion of an improved method, and in the hope that it may lead to a procedure more satisfactory still.*

Though most text-books of Physics deal with the addition of such vector quantities as velocity, force, acceleration, etc., the treatments given in introduction of the addition law are seldom satisfactory. They fall roughly into two main categories: first, those in which the fundamental reasons for the law are either built up from special cases or are dodged entirely; and second, those—usually in books of a more purely mathematical type—in which the law is either *assumed* or is made the matter of a bare definition without any attempt to give a foundation in reason or a connection with the student's previous mathematical conceptions. The result is that the lad either never knows that there is a rational basis, or that he acquires a vague trust in the law only as the result of a general though unanalyzed feeling that somehow it must be all right since its consequences seem to be more or less reasonable.

As one interested in the sound teaching of elementary Physics, the writer feels that such work should be coherent not only with regard to those subjects usually grouped as Physics, but with regard to the mathematical group as well; and that therefore the second of the categories referred to above is, from this point of view, even more objectionable than the first. The introduction of new definitions without discussion of their bases, as a means of evading pedagogical difficulties, is but a form of the universally condemned practice of proposing a new hypothesis to meet each new incoherence of theory. It is very important that the student should see in the law of vector addition—not a fact unrelated or opposed to his previous knowledge of addition—but an extension of the mathematical ideas that he already possesses.

In view of these considerations the following attempt has been made to reach the beginnings of a satisfactory method. The set of notes given below has been put into the hands of freshman classes with encouraging results. The notes are not supposed to be sufficient of themselves but are used with the explanations and comments of the lecturer to develop the required ideas.

Those teachers who find the subject worth considering are requested to examine the plan, not as a string of definitions, but as an attempt to organize certain exact ideas the material for which already exists, though somewhat chaotically, in the mind of the pupil. The boy has never perhaps used the term *vector*, but he is really quite familiar with many quantities of the sort, so that a few moments reflection suffices for him to recognize the existence of the co-factors *magnitude* and *direction*. But just as he always finds more difficulty with the different planes in solid geometry than with plane geometry itself—in spite of the fact that his whole experience has been in a world of three dimensions and not of two—so here, though he has always thought of velocity, of force, etc., in terms of the co-factors just mentioned, it is not the symbols themselves but their combination that gives him trouble. This can surely be overcome by more careful teaching as to the real basis of the addition law.

The plan given below follows Maxwell (*Matter and Motion*, Arts. VIII to XII) in as far as it deals with vectors *as such*, and thereby avoids the bad practice of duplicating the proof of the addition law for velocities, for forces, for couples, etc., etc. The differences from Maxwell are to be found (1) in the "static" idea of the vector as opposed to the "kinetic" or "operational" idea, and (2) in the application of the distributive law, (§4) which as far as the writer knows has not before been consciously employed in this connection. Sections 1 to 3 are prerequisite to 4; and those that follow deal with the idea of the sum, and with the practical manipulation of vectors in their application to simple problems. Section 7, for instance, with its instruction for the separation of "position" and "vector" diagrams, presents a well known method that ought to be insisted on in every text, as it has not only a high pedagogic value in pointing out distinctions where they are essential, but, in virtue of this very fact, prevents confusion and leads to clearer solutions.

The Addition of Physical Quantities.

§1. The specification of a Physical quantity requires at least two factors, (1) a *unit* defining the standard in terms of which the quantity in question is to be measured; and (2) a *measure*, specifying the relation of the quantity to the unit, e. g., 7 days, 9 feet, 5 quarts. The number here is the measure and the name of the unit follows.

The *unit* itself is a purely Physical quantity having definite dimensions, i. e., being built up from the arbitrary fundamental units in a definite way, e. g., a unit of velocity is always a displacement divided by a time. It must be of the same nature as the quantity to be measured, i. e., a unit of length must be a length; a unit of velocity, a velocity; etc., etc.

The *measure* is a purely mathematical quantity that may be either of the two kinds of number; i. e., it may be either a *scalar* or a *vector* (see the next section).

§2. A *scalar* is purely algebraic or arithmetic. It is a pure number. It may be greater or less than another scalar but can have no relation to direction. We represent scalars either by dots or score lines in symmetrical forms, as in the "picture numbers" of the kindergarten, or by more conventionalized systems, as in the Roman or in the Arabic numerals: *In any case the sign used is not the scalar but merely the symbol for it.*

A *vector* is a geometrical quantity involving direction as well as magnitude. The relation of any point in space to a given origin or datum point involves the two independent factors of *direction* and of *distance;* i. e., of direction and of geometrical magnitude. The most expressive symbol for a vector is therefore *a directed line-segment;* the direction of which specifies, on some arbitrary convention (as putting the north to the top in maps), the direction of the vector, and the length of the segment on some given scale specifies its magnitude. *As in the case of the scalar, the line-segment is not the vector but merely the symbol for it.*

§3. Physical quantities fall into two classes according as they require scalar or vector measures. Scalar quantities are those requiring scalar measures, as mass, energy, temperature, etc. 3 grams, 3 foot-pounds, 3 degrees centigrade, are all scalar quantities. The measure "3" in each case being a pure number (a scalar) and is identical in the cases here mentioned. The units are different in each case. Vector quantities are those requiring vector measures, as force, velocity, acceleration, etc. 3 dynes north, 3 feet per second north, 3 feet per second per second north, are all vector quantities in which the measures 3, north are identical vectors in each case, though the units are different.

§4. *The distributive law*, i. e., $ax+bx+cx \quad . \quad . \quad . = (a+b+c \quad . \quad . \quad)x$ is one of the fundamental mathematical laws. It is founded on human experience and no experience contrary to it has ever been found. It therefore stands exactly on the same basis as the postulates and axioms of geometry, the other fundamental laws of mathematics, and the fundamental law of mechanics (F = Ma). It is independent of the nature of the quantities involved.

When we consider the addition or subtraction of Physical quantities we use this law. If, in any case, U be a unit and a, b, c be measures (either scalar or vector) we have

$$a\mathrm{U}+b\mathrm{U}+c\mathrm{U} \quad . \quad . \quad . = (a+b+c+ \quad . \quad . \quad . \quad)\mathrm{U}$$

Thus independently of the nature of the products aU, bU etc. we see that the sum involves the unit, U, and the mathematical sum of the other factors. If these factors be scalars they are to be added according to the scalar law, and if they be vectors they are to be added according to the vector law.

§5. The sum of two scalars is usually defined as a scalar that simultaneously and exactly contains both of the original or component numbers. Thus $2+3 = 5$. This idea of the sum will perhaps be best expressed in the kindergarten "picture numbers" $\cdot\cdot + \therefore = ::\cdot$ Applying the distributive law; 2 days+3 days = (2+3) days = 5 days. This is perfectly familiar, but is introduced to call attention to the meaning of the word "sum" and to prepare for the next statement. Similarly the sum of two vectors may be viewed as a vector that simultaneously and exactly contains both of the original or component vectors. Taking for simplicity the rectangular case shown in Fig. I, and obtaining the sum 5 east+3 north. We note that if *oc* be drawn east and west through the origin and if A B be parallel to it but 3 units north, every point in AB (irrespective of its east or west position) is *3 units north* of O. Similarly every point in CD is 5 units east of O, and the only point that is both 5 units east and 3 units north of O is the intersection E. Thus in the meaning of the term "sum" given in this section the vector represented by OE is the sum of the vectors represented respectively by OC and CE for it simultaneously and exactly fulfils both conditions. This addition may be more simply represented by the three directed lines O′C′, C′E′ and O′E′ in the same figure. The extension to the more general case where AOC is not a right angle is left to the student. That this idea of "sum" is an extension of and is consistent with that of scalar sum will be

at once evident by considering the case where both vectors have the same direction.

Having now the idea of the sum, a more satisfactory establishment of the law is to be obtained as follows:

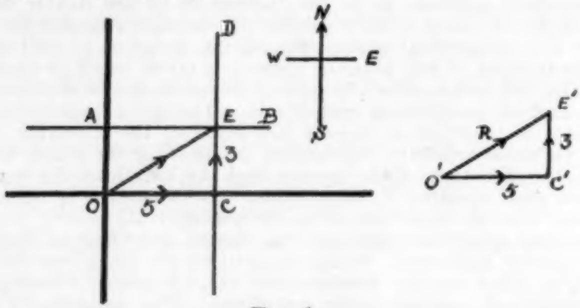


FIG. 1.

Let the vectors to be added be: A = magnitude p, θ above OX, and B = magnitude q, φ above OX. Take the plane of the paper as the plane containing the two vectors. With O as origin, OX as a datum direction, and any convenient length as a unit by which to express magnitude, we find a point C by going from O in a direction θ and above OX for a distance p (see fig. 2). Note that C is thus uniquely determined (in relation

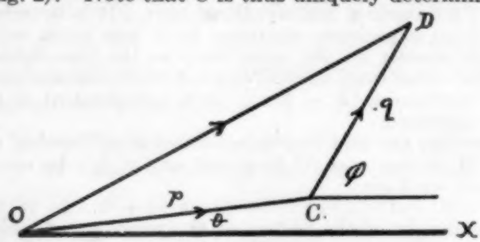


FIG. 2.

to O); that is, by this process we can arrive at no point other than C. Therefore OC represents, with the conventions of direction and unit magnitude just adopted, the magnitude and the direction that specify the vector A. In exactly the same way if we proceed from C in a direction φ above OX and to a distance from C equal to p units, we arrive uniquely at D. But OD represents a definite vector. Therefore a magnitude p in the direction OC *plus* a magnitude q in the direction CD, is equivalent to a magnitude OD in the direction OD.

Since this result is obtained solely by reason of the component vectors having specified magnitudes in specified directions, the result holds for all quantities jointly dependent on these two properties, i. e., the law obtained is valid for the addition of any two vectors. The extension to any number of vectors follows in the usual way.

Note: The student who examines the basis of these proofs will probably be disappointed at finding that he has been thinking in terms of a special vector, viz., displacement. This difficulty will vanish when he sees that it arises from the fact that the *vector symbol*—the directed straight line—is inseparably connected with an actual displacement. He will see that the general conclusion resulting from these proofs (quite apart from the mere symbol employed) has been obtained solely in virtue of the cofactors magnitude and direction and that therefore it is fully justified. The next question will probably be "Can we not attain this end by a less elaborate process?" But it is only through an argument, at least analogous to the one given, that we can separate the general proposition from the particular aspect imposed by the choice of our symbol for a vector.

[1]This is not a mere repetition, as might appear at first sight. Each method develops ideas not obtained in the other.

It is possible, but unlikely, that a symbol could be found that would be free from this objection; but the very defect in our symbol in respect to the mere establishment of the addition law is one of its strongest claims for continued use in the actual manipulation of these quantities.

§6. The student must recognize that in adding the vectors A and B (I, fig. 3) we may use either the parallelogram (II) or the simpler triangle (III), as one is but a complex way of obtaining the result more simply given by the other. It will soon be found that the triangle construction affords simpler and more direct solutions in most cases. It is of the utmost importance to put arrows on the lines (they cause less confusion near the middle of the line) and to note that in their addition we draw the second from the point of the first and not "tail to tail" as in the parallelogram construction.

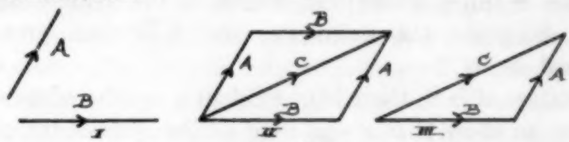


FIG. 3.

§7. In all problems involving vectors it is advisable to make two separate and distinct diagrams (see fig. 4). The first, called a position diagram, gives the lines of action of the vector quantities (velocities, forces, etc.), their relative positions and shows the dimensions, distances, etc., of the problem. The second diagram (vector diagram) is purely one of vector relations. The reason for keeping the two separate is found in the fact that in the first lines they represent distances; and in the second they represent vectors. If the two diagrams are superposed (as is so often done in text-books) confusion is almost sure to result from the two distinct meanings that any given line may have. For instance, consider a mass supported from a beam by two cords (30 ft. and 10 ft. long) as shown in the position diagram in fig. 4. The first diagram gives the dimensions, names the angles, and shows the position of the three forces holding the mass M in equilibrium. The second is the vector diagram giving the addition of these three forces and showing the relations that must obtain

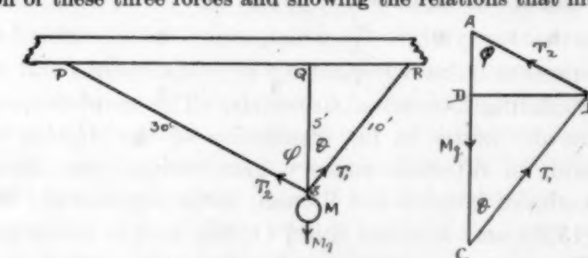


FIG. 4.

for a zero resultant. Note that, as all the lines in the vector diagram represent vectors, they must be parallel to the corresponding lines in the position diagram. Thus the only quantities in common between the two diagrams are the *angles*. So from the positions we have θ and φ known, as the values of their cosines (QS/SP and QS/PS) are known, and these applied to the vector diagram give us T_1 and T_5 in terms of Mg. Had we drawn one diagram on top of the other the line SR, for instance, would at one time have had to represent a length of 10 feet and at another a force of T_1 lb. wt. Confusion would have been invited and would have been almost sure to have ensued.

Keep the diagrams separate and remember that the only quantities common to the two diagrams are the angles.

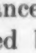
These principles and methods, though used here for a condition of equilibrium (where the resultant is zero) apply in the same way to any case of vector addition. Through the distributive law (§4) they apply to the addition of any type of vector quantity.

HOW X CAME TO STAND FOR UNKNOWN QUANTITY.

By Florian Cajori,
University of California, Berkeley, Calif.

Incorrect historical statements of the manner in which the letter x came to be used as a symbol for unknown quantity in mathematics are widely current. Thus, in Webster's *New International Dictionary of the English Language*, 1914, 1917, we read: "*Math.* An unknown quantity. X was used as an abbreviation for Ar. *shei* a thing, something, which, in the Middle Ages, was used to designate the unknown, and was then prevailingly transcribed as *xei*."

As a matter of fact, there is no evidence, worthy of serious consideration, to show that x was used as the symbol for unknown quantity before the publication of Descartes' *Géométrie*, in 1637. This is the work in which Descartes made known the "analytical geometry," of which he and Fermat are the discoverers. Descartes introduces, without comment, the use of the first letters of the alphabet to signify *known* quantities, and the use of the last letters to signify *unknown* quantities. The symbols for the knowns are introduced in the order a, b, c, . . . Of the symbols for unknowns he introduces z first, later y, and finally x. There is no indication that he followed the practice of earlier writers in this use of x. This symbol came as the third letter from the end of the alphabet, z, y, x.

It is true that the symbol  which somewhat resembled x, was used to represent unknown quantity in several books and manuscripts antedating Descartes' *Géométrie*. This symbol was used by Robert of Chester in his translation of the *Algebra* of Al-Khowarizmi, in fifteenth century manuscripts now deposited in the libraries of Munich and Vienna, in the algebras of Christoff Rudolff (1525) and Michael Stifel (1553), and in other printed books. German writers have interpreted this symbol as being the initial letter r of the words *radix*, *res*, with a small loop attached to indicate the omission of letters. The question has arisen, did Descartes misinterpret this symbol as being the letter x, because of its resemblance to x, and then proceed to use it as the symbol for unknown quantity? J. Tropfke, in his *Geschichte der Elementar-Mathematik*, Vol. 1, 1902, p. 150, advances this view. The falsity of this hypothesis was established by G. Eneström who showed that Descartes, in a letter of March 16, 1619, addressed to Isaac Beeckman, used the symbol  and gave it a form quite distinct from x, hence he later could not have mistaken the symbol for an x.

There is nothing to support another hypothesis, due to G. Wertheim, that the Descartian x is the notation of the Italian mathematician Cataldi who represented the first power of the unknown by a crossed "one," thus $\cancel{1}$.

The early part of the seventeenth century was one of great progress in algebra. Many different suggestions were made in algebraic notation. The great French algebraist Vieta began to represent general numbers by capital letters. Thomas Harriot in England preferred small letters. The notation suggested by Vieta and favored by A. Girard made vowels stand for unknowns and consonants for knowns. As we have seen, Descartes discarded this usage for a new notation of his own. Thomas Brancker, in his translation into English of Rahn's *Algebra*, in 1668, discarded Descartes' scheme and remarked: "Des Carte's way is to signifie *known* quantities by the *former* Letters of the Alphabet, and *unknown* by the *latter* [z, y, x, etc.]. But I choose to signifie the *unknown* quantities by *small* Letters, and the *known* by *Capitals*." However, Descartes' way was the one finally adopted.

These facts indicate that in the period of Vieta and Descartes several different algebraic symbols were arbitrarily selected, without reference to medieval or Arabic usage. Any one reading Descartes' *Géométrie* will see that his manner of introducing symbols for known and unknown quantities seems free from tradition and their choice purely arbitrary.

To what extent the letter x has been incorporated in mathematical language is illustrated by the French expression *etre fort en x*, which means "being strong in mathematics." In the same way, *tete a x* means "a mathematical head." The French give an amusing "demonstration" of the proposition, that aged men who were *tete a x* never were pressed into military service, so as to have been conscripts. For, if one of them had been a conscript, he would now be an ex-conscript. Expressed in symbols, we would have,

$$^{\theta}x = \text{ex-conscript.}$$

Dividing both sides by x, gives

$$\theta = \text{e-conscript.}$$

Dividing now by e yields

$$\text{conscript} = \frac{\theta}{e}.$$

According to this, a conscript would have *la tete assurée* (i. e. θ over e, or, the *head assured*), which is absurd.

EDUCATORS AND CHEMISTS LAUNCH MOVEMENT TO COUNTERACT INSIDIOUS ATTACK OF GERMANY UPON AMERICAN CHEMICAL INDUSTRY.

Educators and chemists have launched a vigorous movement for driving an insidious form of German propaganda from the universities and scientific schools of the United States, according to a bulletin issued yesterday by the American Chemical Society.

They are following the policy inaugurated by the Indiana State Board of Education which has already directed its superintendent of public instruction to issue a letter to all schools under his jurisdiction insisting that every piece of chemical apparatus, and all scientific supplies purchased for educational purposes be purchased in the United States. This action was taken upon the reading of a letter from Harry E. Barnard, State Food and Drug Commissioner of Indiana, in which he pointed out that the supremacy of German-made chemical supplies was still in the ascendant on account of the crafty way in which the Teuton manufacturers had availed themselves of a peculiar clause in the Tariff Act.

Under this provision, American universities, colleges and schools, are permitted to import chemical glassware and porcelain and scientific apparatus free of duty. It was demonstrated in the European War that American manufacturers were fully able to meet this demand. Although they had some tariff protection, the duty-free privileges accorded to educational institutions had the effect of impressing upon students that only vessels and apparatus "made in Germany" were of any real value for scientific purposes. After completing their studies the young chemists were inclined to carry the same idea into the laboratories in which they were employed. As a result, the infant American industries in chemical porcelain and glassware and in scientific instruments, generally, were constantly throttled by German competition and Hun selling psychology.

In the reconstruction of American industry after the War, leading chemists have appeared before the Committee on Ways and Means of the House of Representatives in support of certain bills (H. R. 3734–3735–4386) which are designed to break the German grip upon our chemical industries.

At a meeting of the Council of the American Chemical Society held in New York City, the opinion was expressed that in order to develop domestic sources of supply of apparatus and chemicals, it was necessary that Congress repeal that section of the tariff legislation which gives to educational institutions the privilege of importing such material duty-free. The Council declared its belief that this duty-free privilege has furnished an important medium for foreign propaganda, creating in the minds of the youth of this country an impression of the superiority of such foreign-made material.

Dr. Charles H. Herty, editor of the Journal of Industrial and Engineering Chemistry, appearing before the Committee on Ways and Means, recently, said that he considered the provision with regard to educational institutions one of the most vicious pieces of legislation ever passed.

"Its effect," said he, "is to take away the key from a key-industry. It exercises a most demoralizing influence upon the students of our universities. It opens the door for propaganda and puts the results of German labor and those of Japanese labor, for that matter, before the students every day. As a result the students are being trained in an atmosphere of dependence on industries of foreign countries."

Colonel M. A. Reasoner, officer in charge of the Field Medical Supply Depot of the United States, testified before the same Committee that he had for the last three years purchased laboratory supplies for the entire army. He declared his belief that all such apparatus should be of American make.

A "FLU" DREAM IN MATHEMATICS.

By William A. Austin,
Fresno High School and Junior College, Fresno, California.

One night, while under the influence of the "flu," I had a dream. In my vision I was walking the streets of a large city early one morning. I came to a big building covering a whole square. While trying to determine what kind of a building this was, I observed many boys and girls of high-school age entering it. I was about to conclude that this was a school building when I discovered that it was not yet eight o'clock. I remembered also that it was Saturday,

I walked up nearer the entrance and these young people continued to enter in greater numbers. They were not pupils, surely, for not one of them carried a book or tablet. It was not a factory, either, for the boys and girls were dressed in school clothes. Now and then a well-dressed man or woman entered, also.

My curiosity was so aroused that I hailed one of the young men and said, "Pardon me. Will you kindly tell me what kind of a building this is?"

"This is a high school, sir," he replied.

"But today is Saturday," I said. "You do not attend school on Saturday, do you?"

"Oh, yes, indeed," he answered. "We work till noon."

"Where are your books, papers, and tablets?"

"In the building."

"Do you not take them home to study at night?"

"No, sir; none of us does."

"Well, then, when do you get your lessons?"

"During school hours."

"When do you recite your lessons?"

"Recite our lessons!" he repeated, with a puzzled expression. "What do you mean?"

I was also perplexed, for I thought every school boy knew the tortures of reciting lessons. "When do you have your recitations?" I inquired.

"We have no recitations. We just do our work," he replied.

I was at a loss to know what next to ask. No recitations, no home study, no bundle of books—just work—and yet he called it a high school. So I stammered, "I do not understand."

He smiled at my ignorance and explained, "We begin work at eight and quit at five. We have an hour off at noon for lunch.

We work every day of the week except Saturday afternoons. We do our regular school work during this time; and, well, really, that is about all there is to tell. You might come and see for yourself."

I thanked him and he hurried in.

There were only a few pupils entering now. Soon I heard a gong ring and afterwards all was quiet.

"And he called this a school," I meditated. "He said they *worked* from eight to five. Work in a school! Strange, I must say. No gazing at books; no scribbling on tablets; no reciting of lessons poorly prepared at home. No, not that; it was just work. Very odd, indeed."

I decided to investigate. I went to the entrance and walked in. There was the spacious hallway characteristic of a large school building. It was really free of students! Everything was quiet except the clicking of a typewriter in an adjoining room, which was evidently the office.

In the office I stammered around in the presence of the principal, unable to make my desires clear. He had the air of a man of business. This took me off my feet. I was not prepared for such a man. I was expecting to meet a pedagog.

"You wish to see our work, do you?" he said. "Very well. Just make yourself at home. Go into any of the rooms and remain as long as you like."

After a few brief directions and explanations he excused himself and turned to his duties. I hesitated, thumbed my hat, and backed out of the office.

Strange, indeed! I expected to learn all about the school in the office. I was, therefore, disappointed. I wandered about the halls looking at the cards on the doors. At length I read on a card, "Geometry, Mr. ———, Teacher."

This was exactly what I wished to see. I was to find boys and girls at work, not in a recitation. How could geometry be ordinary work? How could pupils go into a class in geometry with the idea of working, without first having committed a lot of stuff to memory ready to rattle off in a recitation? I was very anxious to see inside.

I prepared myself for the usual ordeal of entering a recitation in a high school. I imagined thirty boys and girls turning and twisting to stare at me as I should step into the room. I prepared to meet the teacher's questioning gaze. I even framed a little speech for him. Thus prepared I opened the door very quietly and squeezed in.

Of all things! Where was I, anyway? Not a single eye discovered me. The teacher did not even see me. I felt embarrassed and thought that I must be trespassing. The teacher was not at his desk, and it was some time before I saw him.

What a strange recitation in geometry. There was no teacher laboring through a visionary proposition at the board, nor trying to pump stuff out of boys and girls. There was no pupil at the board pointing haphazardly at a drawing and rattling off meaningless words. There was no pupil standing reciting.

My presence was still unobserved, and I had time to survey the room, this workshop in geometry. There were no rows of seats and desks. There were no pupils sitting at strict attention. There was no teacher sitting behind a desk with an eagle eye on every culprit. In fact, there were no ordinary desks in the room. Instead, there were several large tables much like drawing tables. There were blackboards on the walls, but on them there were not many drawings nor much writing. On the walls above the blackboards were pinned rows of drawings of all sorts of geometric constructions. A few large colored drawings were framed and hung on the walls. There were also framed pictures of the famous mathematicians, Euclid and Taylor. One or two large drawings of some engineering project also added to the beauty of the room.

Each student was seated at a table and evidently intent upon some bit of work. I stepped up near enough to observe more in detail. Each pupil had a drawing board on which was a piece of paper fastened down with thumb tacks. Each one had a T-square, a small set of drawing tools, and a little book.

I walked along behind a group of these pupils to see what they were doing. No two pupils seemed to be doing the same things. Yet once in a while two or three pupils would come and stand around another student and enter into a discussion about something on the drawing being made or about some point in the little book. Once a pupil went to a bookcase, and, after selecting a book, returned to his seat to study a reference to it.

One boy I stopped to observe had his paper divided into four equal parts by two perpendicular lines. In the first division he had an angle drawn and the constructions made for the bisector of that angle. Underneath the drawing he had written out neatly what he had started with, the angle ABC; what he

was to do, to bisect the angle; and how he did it. This was written about as follows: 1. With B as a center, and with any radius, strike arcs intersecting the sides AB and BC at the points M and N, respectively. 2. With any radius, and with points M and N as centers, strike arcs which intersect at P. 3. Join B with P. 4. This line, BP, is the bisector of the angle.

In the second division he had constructed the perpendicular from a point P to a line AB. Underneath this drawing was the discussion. He was working on a drawing in the third division at this time, but I could see that he was erecting a perpendicular to a line from a point on the line.

I stepped up to see what a girl was doing. She had her paper divided into two equal parts. In the upper half she had made a triangle, and in it she had constructed the bisectors of the three angles. A discussion of her construction was underneath the drawing. In the lower part she was constructing the medians of a triangle.

Another pupil had drawn a large triangle in the upper part of his paper. The three altitudes to this triangle had been constructed. Underneath the drawing the student had almost completed the discussion. After briefly stating what was taken, what was required, and how it was done, he had this additional discussion: 1. The area of any triangle equals one-half the product of the base and altitude. 2. The base AB measures 38.5 and the altitude CD measures 14. 3. The area is $\frac{1}{2} \times 38.5 \times 14$, or 262.5. 4. Base CB measures 31 and the altitude AE measurer 17. 5. The area is $\frac{1}{2} \times 31 \times 17$, or 263.5. The pupil was just then in the act of measuring the base AC and the altitude BF with a ruler. He evidently was checking the area of his triangle with the three bases and the corresponding altitude of each.

Another pupil had made a triangle. At the side of this triangle he had constructed another triangle with two sides and the included angle equal to the two sides and included angle of the original triangle. One could see his lines of construction on his drawing, and the discussion written underneath indicated that this is what he had done. Beneath this was written, "*Proposition:* Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other." Then followed the ordinary proof of the proposition. About the only difference between

his proof and the one usually given in a textbook was the use of the expression "I made it so" instead of the word "Given."

By this time I had wandered around near the teacher who was just then talking to a group of students. They were pointing to a drawing one was making and examining the little book. I could hear the questions they were asking and the replies made by the teacher. On pieces of paper they were making sketches of all sorts. The teacher assisted in making lines on these sketches, asking and answering many questions. Finally, they all seemed satisfied and went back to their own drawings and became absorbed in their work. I had been following the teacher about for some time trying to get a chance to speak to him. At last he looked up, smiled, and came to me. His greeting made me feel most welcome. He soon discovered that I was deeply interested in the work and also that I was very ignorant of the work his pupils were doing. He invited me to take a chair with him at his desk.

"These pupils, as you see," he explained, "are working out their geometry according to directions, and they are learning many things in a way they will not forget soon. This pupil," he said, pointing to a drawing which a boy placed on his desk just that moment, "has an angle, ABC, here. At this point he has drawn intersecting lines, which are constructed, as you can see by these arcs, parallel respectively to the sides of the original angle. This makes another angle, RMN. In Figure II, you see, he has made two angles, one of which has its sides constructed perpendicular, respectively, to the sides of the other. You can see all the arcs in the construction.

"Now, John," continued the teacher, "in this first drawing what did you take?"

"I took the angle ABC," John replied.

"Tell, in order, what you constructed."

"I then made angle 2 equal to angle 1. This makes line MS parallel to AB. Then I made angle 3 equal to angle 2. This gave me a line NR parallel to BC, since angle 2 and angle 3 are corresponding angles."

"What is true about angle 1 and angle 3?"

"Angle 1 equals angle 3, since both are equal to angle 2."

"Now, what construction have you made?"

"I have made two angles whose sides are respectively parallel."

"What have you done in this second drawing?"

"I have made two angles whose sides are respectively perpendicular."

"What can you assert about two such angles?"

"They are equal."

"What, therefore, is your general proposition?"

"If the sides of an angle are parallel or perpendicular, respectively, to the sides of another angle, the two angles are either equal or supplemental."

"Well, can you not write this all out?"

"Yes, sir."

"Can you also write out the proof?"

"Yes, sir."

"Well, John, what is it, then, that bothers you?"

"The proposition states that the angles are either equal or supplemental," said John. "I do not see how they can be supplemental."

"That is easy," said the teacher. "Is angle 3 the only angle made by the two lines MN and NR?"

John thought a moment and replied, "No, sir."

"Show me another."

John took his pencil and pointed to the angle SNR.

"Mark this angle 4," said the teacher.

"Oh, I see, I see," John exclaimed. He snatched the drawing from the desk and hurried back to his table.

An excellent recitation, I thought. And it was evidently conducted for my special benefit.

"These students seem to enjoy their work," I ventured. "It seems very odd to me to see pupils enjoying geometry."

"Well, you see," explained the teacher, "they are doing things with their hands. High-school pupils are happiest when their bodies are active. You must remember that these pupils are chiefly muscular and impulsive in their habits at this age. We are merely appealing to their natural instincts. You can reach high-school pupils' minds more effectively by means of their muscles than through the sense of sight or hearing. You may require a pupil to gaze at a drawing and the demonstration in the text or on the board, or you may have him listen to explanation and discussion by teacher and pupil, but unless he does the thing for himself it has little significance to him. A man, as well as a pupil, remembers what he does better than what he sees or hears. When he makes the construction for himself he sees each step in the process in its logical order; but when he sees the completed drawing in the book or on the board, he cannot distinguish the beginning from the end, and he is therefore bewildered.

"Children at this age are no different from the rest of us," he continued. "They think in terms of the concrete. They obtain conclusions from a study of many related facts. Inductive thinking, rather than deductive thinking, is their method or reasoning. We propose that they make a certain round of active thinking. First, they think through their particular drawings and measurements to a general truth; second, they state the general proposition; third, they demonstrate the truth of this proposition which they have thus obtained (not obtained from a book by memory); and, lastly, they solve problems by means of the proposition just proved.

"According to former methods of teaching geometry, a general proposition, thought out by mature minds and formulated into correct English, was thrust upon the pupil and he was expected to comprehend it and go through the logical argument in demonstrating it. These theorems and their proofs, complete and perfect in every detail, were fed to pupils whole and in large doses. But the truth of the proposition seldom dawned upon the pupil's mind at all once and hence he was helpless. They were, therefore, obliged to commit to memory in order to make a good showing in the recitation. By our modern plan the pupil sees these things in their unfoldment as a process and not as a completed whole.

"Pupils delight in informality in a schoolroom. These are not recitations in which formal discipline and concert thinking are required. Our pupils are happy at work. Moreover, pupils desire to understand what they are doing and to see its application and its usefulness. These things are not evident to them in the beginning."

"I had not thought of it quite in that light," I confessed, after trying to follow the sound pedagogy of his discourse. "Another strange thing to me is the fact that no two pupils seem to be doing the same things."

"That is not strange," he said, "when you stop to realize that no two men will be doing the same things at any given moment, after they have been working on the same task for some time. Some are faster and more active than others. Some are ambitious, others are indifferent, and still others are really lazy. In this work each pupil is given the opportunity to show his individuality. And that is just what we desire."

"But they do not all complete the work at the same time," I said.

"True enough. But that leads to no harm. In fact, each student has that much more of an opportunity to show what is in him. They all have a certain amount of work to do. When they complete that bit of work and it has all received my O. K., they receive their pay. Pupils have completed this work satisfactorily in less than half a school year, while others have taken over a school year to do the same work.

"As soon as a pupil completes these plates," he explained, as he picked up a bunch of them from the desk before him, "they are handed in. I go over each one. If the drawing is correct and the discussion is good enough, I stamp it as satisfactory. The pupil then keeps all these plates until he has completed the required number. I then estimate the worth of his work and enter his grade for geometry in the office. If the plate is not satisfactory, I hand it back with my comments for it to be corrected or done over again."

"What text in geometry do you use?" I inquired.

"We use no text," he replied. "This little book, *A Manual of Geometry*, is the guide for the work of the pupil. On the shelves there," he continued, pointing to some books in a case in the corner of the room, "are copies of many of the best texts in geometry. We have several copies of some of the best texts. These are library books used by pupils as references. Their manual directs them in making constructions, proofs, and in the solution of exercises. You have observed pupils using those books freely. You might be interested in an examination of this manual."

I took the manual and commenced a careful examination of it.

"By the way," the instructor said, after picking up a few plates made by students, "you might be more interested in the manual if you examine some of these plates in connection with the corresponding directions for them given in the manual."

These plates were so neatly done in connection with the work outlined in the manual that I made a copy of several of the plates and of the directions for their construction and demonstration. The following will give an idea of their work:

Plate XXIX. Definition: A parallelogram is a quadrilateral which has two pairs of opposite sides parallel.

Figure 1. Draw any two intersecting lines, AB and AD. At point B, construct a line parallel to AD. At point D construct a line parallel to AB. Let these two lines intersect at C. What kind of a figure is ABCD? Why?

Figure II. Draw any two intersecting lines, AB and AD. Construct at D a line parallel to AB. Make CD equal to AB. Join C and B. What kind of a figure is ABCD? What condition of construction makes

this quadrilateral a parallelogram? Will every quadrilateral constructed in this manner be a parallelogram?

Proposition: A quadrilateral is a parallelogram if one pair of opposite sides are both equal and parallel.

To prove this proposition draw the diagonal DB. Mark angle at A, 1; the exterior angle at D, 2; the angles at B, 3 and 5; and the angles at D, 4 and 6. Prove triangle ABD congruent to triangle BDC, using two sides and the included angle. What other angles does this prove equal? Therefore, what other two lines are parallel? Now apply the definition of a parallelogram.

Figure III. Draw any two intersecting lines, making each a given length. Call this angle BAD. Make an arc at C with center at D and radius equal to AB. Using AD as radius and B as center, strike an arc intersecting the first arc at C. Join D and C, and B with C. What kind of a figure is ABCD? Will every quadrilateral constructed in this manner be a parallelogram? What condition of construction makes this quadrilateral a parallelogram?

Proposition: A quadrilateral is a parallelogram if it has two pairs of opposite sides equal.

To prove this proposition draw a diagonal, say BD. Can you prove the two triangles ABD and BDC congruent? By what proposition? What angles at D, therefore, equal angles at B? Since these sets of angles are equal, what sets of lines are parallel? Why? Now use the definition of a parallelogram and draw your conclusion.

Plate LVII. Construct three right triangles, calling each ABC, but differing in lengths of sides. In each triangle construct the perpendicular from the vertex of the right angle C to the hypotenuse AB. Call this perpendicular CD in each. In each triangle, measure CD, and the two segments of the hypotenuse, BD and AD, cut off by the altitude. In each triangle compare the product of the two segments of the hypotenuse, BD and AD, with the square of the length of the perpendicular. What appears to be true? Do you think exact measurements would give exact results? Do you think this will always be true?

If the square of any number equals the product of two other numbers, what is the name given to that number? See definition of mean proportional or geometric mean given in Paragraph 238, page 166, in ———

Proposition: The perpendicular drawn from the vertex of the right angle upon the hypotenuse in a right triangle is the mean proportional between the segments of the hypotenuse cut off by it.

In the proof of this show that the two triangles CDB and CDA are similar by getting two angles of one equal to two angles of the other. Use the fact that two angles are equal if two sides of one are perpendicular respectively to the sides of another. In these two similar triangles pick out the proper proportion among the corresponding sides so that the product of the extremes equals the product of the means, gives the desired fact; i. e., $CD^2 = BD \times AD$.

Exercise 73. The hypotenuse of a right triangle is 50 and the length of the altitude upon the hypotenuse is 24. Find the segments of the hypotenuse made by the altitude.

Plate LXIII. Draw a circle, O. Select a point, P, outside the circle. Draw three secants from this point, P, one of which passes through the center, O, of the circle, and a tangent from point P, tangent at the point G. Let the secants cut the circle at points A and B, C and D, and E and F.

Measure the whole secant BP, and the external segment, AP. Measure similar lengths on the other two secants. Also measure the length of the tangent line. For each secant find the product of the whole secant and the external segment. Also square the tangent line. Compare these three products and the square of the length of the tangent. What do you think is true?

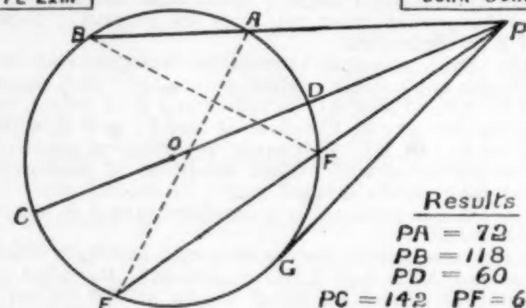
If the product of several numbers, two at a time, always equals the same result, we say their product is a constant.

Proposition: If secants are drawn from an external point to a circle the product of the whole secant and its external segment is constant.

Note: The tangent may be considered a special case in which the whole secant and its external segment are equal, the two points in which it cuts the circle being coincident.

Proposition: The tangent from a point to a circle is the mean proportional between the whole secant and its external segment drawn from the same point.

PLATE LXIII — John Jones



Results
PA = 72
PB = 118
PD = 60
PC = 142 PF = 62
PE = 135 PG = 92

1. $PA \times PB = 8496$ 2. $PD \times PC = 8520$
3. $PF \times PE = 8370$ 4. $PG \times PG = \overline{PG}^2 = 8464$

Proposition: If secants and tangents are drawn from an external point to a circle, the product of the whole secant and the external segment equals the square of the tangent, or a constant.

Given: Secants PB, PC, PE; tangent, PG.
To prove: $PA \times PB = PF \times PE = PD \times PC = \overline{PG}^2$
Proof: 1. Draw AE and BF, forming $\triangle PFB$ & $\triangle PAE$.
2. ∠PPA is common to both triangles.
3. ∠PEA = ∠FBP; Each equals ½ FA.
4. ∴ $\triangle PFB \sim \triangle PAE$. 2 ∠s of one △ 2 ∠s of the other.
5. ∴ PE : PB = PA : PF. Corresponding sides.
6. ∴ $PB \times PA = PE \times PF$.
7. Similarly $PC \times PD = PB \times PA = PE \times PF = \overline{PG}^2 = K$.

Exercise 79. Given PG = 18 and PD = 6.
To find OD, the radius of the circle.
Solution: Let OD = R, then PC = 2R + 6
and $6(2R+6) = 18^2$ or $2R + 6 = 54$
or $2R = 48$ or $R = 24$ Ans

Exercise 80. Given PB = 12, PA = 8, OC = 5
To find PD.
Solution: Let PD = x then PC = x + 10
and $x(x+10) = 8 \cdot 12$ or $x^2 + 10x - 96 = 0$
whence $(x+6)(x-6) = 0$ ∴ $x = 6$ Ans.

In the proof of this proposition, join B and F, and also E and A. This forms two triangles, PBF and PEA. These two triangles can be shown to be similar by getting two angles of one equal to two angles of the other. A certain angle is found in both triangles. Another angle in each is measured by the same arc, FA. Since these triangles are similar their corresponding sides are in proportion. Select the proper sides which will form a proportion giving in the product of the means equaling the product of the extremes the fact that BP×AB equals EP×FP. Since these are *any* two secants the same fact might also be shown about any other two secants. Join G with F, and G with E, and show by a

similar process that GP^2 equals $PE \times PF$. So, in general, we can say that $PA \times PB = PD \times PC = PF \times PE = PG^2 = K$, a constant.

Exercise 79. The distance of an external point to the nearest point on a circle is 6 inches, and the length of a tangent to the circle from this point is 18 inches. Find the radius of the circle.

Exercise 80. The radius of a circle is 5 inches. The length of the whole secant from an external point is 12 inches and the length of the external segment is 8 inches. Find the distance of the external point to the nearest point on the circle.

Plate LXXI. Construct a triangle whose sides are 5, 8, and 12, respectively. Mark the angle opposite the longest side, C, the angle opposite the shortest side, A, and the other angle, B. Bisect the interior angle, B, and let this bisector intersect the side AC at D. Bisect the exterior angle at B, and let this bisector intersect the side AC extended at E.

Measure AB, BC, DA, DC, EA, and EC. With these lengths test this proportion, AB : BC = DA : DC = EA : EC. Does this appear to be a true proportion? Do you believe the truth of this proposition depends upon the accuracy of the construction and the accuracy of your measurements?

Now, AB : BC is the ratio of two sides of that triangle. DA : DC is the ratio of the two segments of the third side of that triangle made by the bisector of the interior angle. EA : EC is the ratio of the two segments of the third side, AC, made by the external division of the third side by the bisector of the exterior angle.

Proposition: The bisectors of the interior and exterior angles at any vertex of a triangle divides the opposite side internally and externally into segments whose ratio equals the ratio of the two sides including that angle.

To prove this proposition draw through C a line parallel to BD, cutting the line AB extended at N. Draw also a line through C parallel to BE, cutting the line AB at M. Mark the two interior angles at B, 1 and 2; the two exterior angles at B, 7 and 8; angle NCB, 3; angle BNC, 4; angle CMB, 9; and the angle MCB, 10.

Now, BD is parallel to the base, CN, of the triangle ACN. Hence, we get the proportion, AB : BN = AD : DC. For what reason? But BC can be substituted for BN in this proportion, provided BN equals BC. We can get BN equal to BC if triangle NBC is isosceles. Now, angle 3 equals angle 4, since angle 1 equals angle 4. Why? Angle 3 equals angle 2. Why? Angle 1 equals angle 2. Why? Therefore, BC equals BN. Why? Substituting BC for BN in the proportion above we have AB : BC = DA : DC.

In a similar manner the line MC, being parallel to BE, the base of the triangle ABE, the proportion AB : BC = EA : EC can also be obtained. Putting these two proportions together, we obtain the desired proportion AB : BC = DA : DC = EA : EC.

Exercise 82. The three sides of a triangle are 84, 63, and 49. Find the point where the bisector of the interior angle intersects the side whose length is 49. Also find how far the side whose length is 49 must be extended to meet the bisector of the exterior angle.

Having examined the manual and the plates, the thought of the recitations came into my mind. "Do you have no recitations, no discussions, no demonstrations at the board at all?" I inquired.

"Oh, yes, indeed," he replied. "Geometry would not serve its entire purpose if it did not teach a boy or girl to stand before a body of students, state a truth, and then after stating what is given and what is to be done, proceed by logical steps of reasoning to demonstrate the truth of the statement. But we find

that the student is unable to do this at first. He must know a great many facts and have done many things concretely before he is able to stand and talk with any degree of satisfaction. One requirement of each pupil is that he be able to come before the class without paper or notes of any kind, state his proposition or exercise, make his construction free-hand at the time he is discussing the things given and the points to be proved or obtained. Then he is expected to go through a correct line of argument, using his construction, to demonstrate his point or to obtain his result. We have discovered that after a student has done a number of these plates, which you now see them doing, they somehow gain an ability in formal demonstration. We have many original exercises, some of which require rigid proofs and some which require calculations depending upon propositions previously learned. These exercises are assigned for the very purpose of formal demonstration, and frequently we stop our work and take up one or more of these exercises. I wish we had the time," he said, looking at his watch, "I would call for a few of these now."

I looked at my watch and was amazed to see that it was nine-fifteen. What! An hour and fifteen minutes! And here were thirty boys and girls working away all this time on geometry. I could not believe it.

"How long do these pupils work at this stuff?" I inquired.

"Our period is an hour and twenty minutes in length," he replied. "Then they have ten minutes in which to rest, chat a while, and pass to their next class. You see," he continued, "ten minutes is quite sufficient for the pupil's mind to readjust itself so that it may take up a new line of thinking with little effort in the next class."

"Then you have an eight-hour day, do you?" I inquired.

"Oh, yes; why not?"

"But teachers usually become very weary and worn out with even a five-hour day of recitations," I said.

"Indeed, I can remember the day when teachers, both men and women, were completely exhausted at the end of a term's work of recitations such as they had then. Pupils, too, were only too happy when that short day was over. But that time has passed. We have eliminated the traditional recitation. Our school is a workshop, not a place for rigid discipline, order, attention, and reciting. We have removed the nervous tension and strain which were necessary in conducting the old-type

recitation. It was the constant mental effort on the part of the teacher to maintain discipline, attention, and logical thinking in each member of the class which exhausted her energy. Our teachers and pupils come from their eight-hour day now much fresher and happer than they used to when there was a much shorter day."

Another thought, that of tests and examinations, came to me. I inquired about these.

"Well," he replied, "these formal demonstrations, of which I spoke a while ago, serve as one form of tests. And in addition to these there are many plates provided by the manual, such as this one." He handed me Plate XXXVI, which a student has just finished and handed in. "These three original exercises are given there in the manual," he explained, pointing to the page in the book. "Such exercises as these contain few or no suggestions of any kind, as you see. The pupil is expected to prepare this plate with little or no help. These plates come quite frequently, and they serve as good tests for the student's ability. Just examine this plate and these exercises."

Here are the three exercises:

Plate XXXVI. Exercise 57. The lines joining the mid-points of the four sides of a quadrilateral, taken in order, form a parallelogram. Hint: Draw one or both diagonals.

Exercise 58. Two observers, A and B, are 24 miles apart. The observer at B locates an enemy battery due north. The observer at A locates the same battery due east. A third observer goes along the line AB from B a distance of 8 miles, and at that point he locates this same battery along a line perpendicular to the line AB. Find how far each of the three observers is from the battery.

Exercise 59. (1) How many sides has the polygon if the sum of its interior angles equals the sum of its exterior angles? (2) If the sum of the interior angles is two and one-half times the sum of the exterior angles? (3) If the sum of the interior angles is a times the sum of the exterior angles? The answer to this third part is $2(a+1)$.

I also made a copy of the plate containing these three exercises which he handed me.

At this point the gong rang sharply. I instinctively jumped aside into the corner of the room to get out of the path of the students rushing from the room, for I have had serious experiences in the past in attempting to stand in the way of the mad rush of disgusted students from a recitation in geometry. But I was again surprised. No one seemed in a hurry. The students carefully and deliberately put their work away in the same spirit that they had manifested during the whole period and walked out of the room.

BIOLOGY AND THE WAR.

The Great World struggle now so happily terminated called forth the most complete and extensive mobilization of the resources of the world that any age has yet witnessed. Few, if any, storehouses of nature have been left undisturbed. Few of the fields of human endeavor but subserved the war program.

Biologically, *life*—plant, animal, and man—was mobilized, or, if not actually assembled, was inventoried and held in readiness for conscription to prosecute the war to a successful conclusion. No such draft upon the life-resources of the earth for warring purposes has occurred hitherto.

Mobilization! Mobilization!! Mobilization!!! The mobilization of men has resulted in assembling vast armies of men for war purposes; secured for their equipment as fighters the material wealth of the nation—the raw materials, as well as the finished products.

All this has necessitated the floating of huge loans of money to make available the ready cash to transact the stupendous military and business enterprises incident to mobilization and war. Most insistent have been the demands upon the arts and sciences to consummate the program upon which the nation embarked to free the world in a larger and better sense than at any previous period.

The mobilization of men in the various combatant countries, the numerical strength of which mobilization was greater than the combined populations of one-half dozen of the world's greatest cities, stands as one of the marvels of this epochal period. Statistics on army, navy, and aerial man power testify to the draft made upon populations, particularly in the combatant countries of Europe.

Now, if to this great host of front-line participants are added the great armies of workers, male and female, engaged in the primary industrial war activities, and, finally, are added the civilian populations at home who modified their usual practices, or who were specifically organized to contribute directly or indirectly to the furtherance of military endeavor, one may say, that in a very vital sense, *human life* was heavily conscripted.

The mobilization of the material wealth has been equally pronounced. Field, forest, and mine have yielded the raw materials, and the home, mill, and factory use of these sent forth the finished product with an amazing speed. The utilization of all this wealth has been with a prodigality which free

peoples have cheerfully given that the triumph of principles embodying a larger freedom might be dominant the wide world over. Biologically considered, the conservation of the world's material riches has been, in this now passing crisis, a prob em in *utilizing* the life-resources for the *greatest good.* What may have seemed like threatened impairment in certain lines of resources during the progress of the struggle, may, more readily than at this close range it is possible to see, prove the wisdom of making the present extraordinary conscriptions upon all forms of life. Providentially does it begin to appear that the struggle cost perhaps far less than now had it been deferred.

The mobilization of huge sums of money has been carried out in a typically colossal fashion characteristic of the mobilization in general. Statistical studies of the appropriations and expenditures of the various combatant nations reveal that the term, billion, has come to be as common in the consideration of individual items or in the totals of the financing as was the term, million, in normal times. A billion-dollar Congress for the United States, for example, seems puny alongside the actual and proposed Congressional appropriations during 1917 and 1918, which suddenly increased 120 per cent. An examination of the increase of indebtedness forty-seven months after the beginning of the war shows 1,000 per cent, and in a few instances even more increase over pre-war indebtedness. As the debt before the war was measured in millions of dollars, it is now measured in billions!

The contributions of the arts and sciences to the Great War have been of a paramount character. Not only has wide use been made of the past achievements of the arts and sciences, but man has been keenly sensitive all through this period to the possibilities for creating new inventions as well as methods for their control. Mobilization of all that art and science could offer which was practicable to use, and a most intensive study of difficulties to bring new utilizations and new methods to the front marked this period of strife as nothing else did. Perhaps it is not too much to say that the stimulus given to the arts and sciences as fields for human endeavor, caused by the necessity for their exploitation to further war activities, has resulted in greater progress in their advancement than has occurred in many previous decades.

When the final chapter of this momentous struggle shall have

been chronicled and some perspective secured with reference to evaluations, it will then appear, in my judgment, that the biological sciences, together with other fields of science, have taken a most conspicuous and valuable part in the rendition of services. The services of biology are essentially constructive, and found their war application in such fields as physiology, anatomy, hygiene and sanitation, bacteriology and immunology, foods and nutrition, forestry, agriculture, etc.

The *fittest* are called to war. The best physiological specimens of life are chosen for the arduous tasks, the heroic sacrifices. The selective draft law was in its application a selection of the *biologically fit* individuals.

Individuals selected must not only be anatomically sound, but functional disturbances, as well, must be eliminated. The standard was a high one. Individuals with certain defects were carefully studied, and, if not rejected, were treated most systematically to make them efficient specimens. Many physical defects were found and therefore corrective work had its place in this constructive program. Flat feet, defective teeth and eyes, faulty carriage, etc., were some of the problems with which the military organization through its medical corps had to cope to raise the percentage of physically fit.

To keep the individual physically fit was an equally important task. The body must function in keeping with its anatomically sound condition. What a biological principle is here involved! To teach men to take care of their bodies was no mean task. One of the outstanding features of this physical care was the regulation and maintenance of the dietary. In camp life this soon becomes satisfactorily arranged but with a mobile army it presents a problem of grave concern. To keep physically fit, attention must be given, therefore, to the diet for proper functional reactions. The body must be kept clean, too. The bathtub at the front was an element of great value in this connection. There must be strict attention maintained to definite and strenuous daily exercise, and all supplemented by the development of a consciousness of *fit* appearance.

Further, to keep the individual physically fit his surroundings had to be kept sanitary. Garbage must not accumulate; there must be good drainage for the camp; and the air and the water supplies must not suffer from contamination.

The individual was conscripted then, other factors waived for this consideration, who was biologically a good specimen,

and subjected to a rational program of treatment to secure a better bodily machine, supplemented by a program of feeding and exercise which would insure the proper physiological reactions, and, lastly, a most rigorous care-taking of the environmental factors which mean so much in the development and healthfulness of human life. What a barrier was thus builded around each life to protect it from the ravages of disease, the barrier of natural resistance!

To efficiently conserve this natural bodily resistance from attack of infectious germ life, the science of bacteriology stands in a peculiar relation to this problem. All that this science had wrought in the study of plant and animal microbes and parasites which cause decay, fermentation, and disease were focused upon methods of preserving the food, purifying the water, and developing pure culture serums and vaccines for immunization of the body from some of the germ diseases, particularly smallpox, typhoid, malaria and pneumonia, and the treatment of wounds by covering them with paraffine until such time when they could be carefully dressed, or the use of the now famous Dakin's Fluid, discovered by Carrel, which is strong enough to kill germ life but against which the cells of the body can protect themselves by neutralizing it with the sodium contained in their fluid. Thus the natural physiological processes to heal and repair the injured parts were so stimulated and conserved that statistics show that about 95 per cent recovered from wounds, and that the healing was characterized by its rapidity. What a contribution the science of bacteriology has made to minimizing the work of the invisible enemies of functional activities! Medical progress was directed to counteracting the weapons of warfare by the employment of preventive rather than curative methods. The two great weapons of the triumph of medical science have been sanitation and prophylactic inoculation.

From whatever field of biology one examines the contributions, the fact becomes more and more emphasized that the biological sciences have been foremost in the world's greatest endeavor.

The knowledge of animals and animal life, supplemented by studies in connection with the pressing needs of the time, has made a distinctive contribution. A few of these may be mentioned, such as protozoa in relation to disease; the great carriers of disease—fleas, flies, mosquitoes, and lice; also, the habits of

rats, mice, and roaches in relation to filth and disease. The utilization of animals for food, for beasts of burden, for messengers, as the pigeon and dog, for supplying leather, medicinaland other products, was dependent quite primarily upon the systematic and invaluable work of the zoologist.

Plant life and plant uses have been a science no less valuable in the great program of mobilization of resources. The adaptability of different woods for structural purposes, particularly walnut, spruce and pine; the drug, dye, and toxin producing plants; and the currying of the earth by the botanist to find the types of plants yielding substances to supply some particular shortage, or to be used in some new situation which arose because of the war, are a few examples of the services of botanical knowledge.

Agriculture has been busied with quantity production. The seed bed had to give forth its maximum as never before; the insistent demand for the products of the farm—cereals, meat, fat, and vegetables—together with the studies of possible substitutes to take the place of those the demands for which have been greater than the supply, or for those which were not easily transferred, were the lines along which most effort was directed. All of this gave an impetus to agricultural science, which, it is generally believed, will carry over into peace days and thus materially aid in the restoration of the price level of soil products. The whole world has been stimulated by a new realization of the relationship of agriculture to national welfare and security.

The study of the mental characteristics and reactions may be incorporated here as an interesting part of the selective draft program to secure the best life, the most peculiarly adapted individuals for the respective branches of the service. The general characteristics of personal action and the conditions of effective reaction to new situations; the individual differences and their military exploitation for leadership and other lines; the learning process; and the sources and the critical estimate of information were a few of the problems concerning nervous phenomena receiving attention in arriving at a just estimate of the *fit*. Some interesting data are at hand concerning the effects of war on the nervous system, and already such terms, as, "war psychosis," "fear neurosis," "hypomania" are becoming prevalent to describe certain of the effects upon the nervous system of the recruits, and also upon the nervous system of the ones at home.

The rehabilitation of disabled soldiers, sailors, and marines as a post-war program upon the part of the War Department, Bureau of Medicine and Surgery, the Navy Department, and the Federal Board for Vocational Education may be pointed out as one of the most serious attempts the government has ever made to fit these handicapped soldiers for useful civilian activities. The blinded, the deafened, the tuberculous, and mentally "shocked" are alike to be treated, and, in so far as possible, made fit.

One of the most unique biological contributions of which one may think in connection with the progress of the war is that of camouflage. "Camouflage" in Nature has given primal lessons in the art and science of camouflage in war. Sympathetic coloration and mimicry have for a long time excited much interest and extended observational studies to the student of adaptations. The most diverse forms of animal life exhibit most effective sympathetic colorations, for example the inhabitants of regions of snow are white, while those of the desert lands are yellow, and the surface fauna of the sea are transparent. The Teutons and French settling down to trench warfare after the first battle of the Marne began camouflaging their positions. The use of camouflage became very widespread as the war progressed. Upon the sea, as upon the land, much shading, screening, countershading, and splotching of war materials in position or in transit were used, with a high degree of success.

Biology, the study of life, has come to be a science of great service. In peace time or in times of great cataclysm, the study of *life* and its manifestations in this world of environment for a better understanding of the problems of relationship and control of *living* has become the supreme goal. More and more is there need, too, of cooperation. Community, state, national and international cooperation are needed to lessen the enormous wastes, losses and damages which, if unchecked, will sooner or later, boomerang-like, make intolerable the living conditions of generations yet unborn.

NECROLOGY COMMITTEE.

If any person knows of the death of any member of the Central Association of Science and Mathematics, during the past year, will he kindly communicate the facts to the Chairman of the Committee, Mr. A. W. Cavanaugh, Lewis Institute, Chicago, Ill., so the facts may appear in his report.

SCIENCE TEACHING IN THE GRADES.

By M. J. Phillips,
Peabody High School, Pittsburgh, Pa.

During several years of high-school teaching, in biological sciences, I have observed with much interest the enthusiasm of grade pupils for sciences when they entered our classes. The lack of interest has been so evident that a quiet investigation of conditions in the grades has been made. The investigation has occupied much time. The following conclusions seem to be a fair summary:

First, the average grade pupil sees very little of the out-of-doors. He therefore knows very little about it, and cares less.

Second, his science work is human physiology during 6b, 6a, 7b, and 7a years. He calls it "Physiology." Several of the textbooks found in the hands of 6b and 6a pupils have been examined in particular. Bear in mind that these pupils are ten, eleven, and twelve years of age. These pupils were studying the following subjects: Foodstuffs, dietetics, fermentation, mechanics and chemistry of digestion, enzymes, osmosis, vitamins, cookery, gravity, histology of bones, of muscles, of blood, of kidneys, of nerves and nerve centers, stimuli, impulses, light, sound, parasites, bacteria, cultures, tubercles, infection, immunity, smallpox, erysipelas, tetanus, hospitals, poisons, snake bite, and many other equally unsuited and deadening subjects to eleven and twelve-year-olds.

It is unnecessary to even wonder whether these pupils will enjoy such "education" or whether their nerves will tingle from the lively point of contact with their daily interests, or whether they will rush to the out-of-doors at the first opportunity to see these things for themselves, or whether they will come in during the spring, summer, and fall days with bits of Nature they have been observing and desire to learn about. It is even not to be thought of that these pupils should read the "Secret Garden" as a result of their science instruction.

The writer has discovered that very many of the grade teachers permit the spring and fall days to go by without directing the attention of pupils to the early flowers, the birds, etc.

It is needless to enumerate more of the almost unthinkable failures of these teachers, who, by the way, have our children during the years which, if rightly used, could make their whole after lives full of genuine enjoyment of the world of Nature about them.

They may, if city teachers, contend that it is difficult to take pupils to the "country," as city folk are wont to say. If they would but understand that every yard, every lawn, the parks, even the streets, the foliage all about them, offer plenty of material for study.

No argument is needed to convince these teachers that culture may manifest itself in an enjoyment of these out-of-door things, in a habit of seeing things about them, in a fund of information about their habits of life, just as much as in music, pictures, books, and travel, it is to be hoped.

A well-known high-school principal recently in the hearing of the writer lamented the small number, comparatively, of high-school students taking work in science, i. e., general science, botany, zoology, chemistry, physics, and physical geography. It is disappointing when one considers the equipment of the modern high-school laboratory and the requirements of men and women that fit them to do this work. It is further disappointing when one considers that we are living in such a wonderful scientific period.

The grade teacher of the sixth, seventh and eighth years is in no small sense to blame for the situation. Boards of education and school executives who select textbooks as the above, so eminently unsuited to the pupils' needs, certainly deserve much condemnation for such short-sightedness.

A CHEAP AND SERVICEABLE VENTILATING HOOD.

By G. M. Lisk,
Normal School, Alva, Oklahoma.

The awakening of an interest in the subject of chemistry seems to be one of the educational effects of the late war, and we may look for an increasing number of secondary schools to be adding this course to their curricula; since in many cases it will be necessary to "work over" an ordinary recitation room into a chemical laboratory, the question as to whether or not this can be fairly easily done will determine the fate of the chemistry course in many schools.

Next to the necessary plumbing, probably no feature of the equipment will seem to offer more obstacles than that of a suitable ventilating hood. We seem to be possessed with the notion that this part of the laboratory outfit involves a complicated system of pipes, fans, etc., whereas it is believed by the author

that the scheme here outlined will serve the ordinary high school in a very satisfactory manner, and at a minimum cost.

Simply build out the sill of one of the laboratory windows to any desired depth, say eighteen inches. Box up the sides and top; put in a second *inside* window, the lower half raising *behind* the upper half to prevent the likelihood of the escape of gases between the two window halves. This is the whole story and, simple though this arrangement is, the following desirable points are claimed for it:

(1) It costs but little and can be installed in almost any room by local talent.

(2) It is roomy, well-lighted, and easy of access, facts rarely true of the ordinary hood.

(3) By lowering the upper outside sash, we get an outlet for light gases; by raising the lower outside sash, we provide for heavy gases. (In this latter respect it is a distinct advance over most hoods.)

(4) While probably not necessary in most cases, artificial draft can be easily provided for by the simple expedient of hanging from the top a large kerosene lamp, unless it is desired to fit the hood with a gas burner or electric fan.

NEW FOREST SERVICE PHOTOGRAPH EXHIBITS.

New photographic exhibits on "Forestry and Nature Study" and "Farm Woodlands" may now be borrowed from the Forest Service, United States Department of Agriculture, by schools and libraries. The "Forestry and Nature Study" exhibit is a pictorial story of how trees grow, and of the buds, leaves, flowers, and fruits, the typical forms of trees, the different kinds of forests and the influences that affect their growth, and the enemies and friends of the forest. The "Farm Woodland" exhibit, which is especially adapted for use in agricultural and rural schools, shows different types of woodland, how the farmer can use the woodland and sell the product, and how trees make waste land profitable and help the farmer in other ways. The exhibits are made up in panel form, each panel consisting of four sepia enlargements.

Teachers who are interested in the forests in a more general way will find what they need in the original photograph exhibits of the Forest Service, which show forest conditions in the United States, how the forests are used, and how they may be preserved.

For classes in manual training and the like there are exhibits of commercially important woods of the United States with explanatory charts and tables.

Schools that have a lantern or can provide one may borrow sets of lantern slides with prepared outlines for lectures on many topics connected with forestry. For instance, there are sets on forestry in the United States, and on nature study, botany, manual training, geography, and agriculture in relation to forestry, and on street trees and windbreaks. Recently a set has been made up on recreation in the National Forests. Lists of subjects and other details may be secured on application to the Forest Service, Washington, D. C.

TEACHING A "READING" TEXTBOOK OF BOTANY.

BY N. M. GRIER.
Sorbonne School Detachment, A. E. F.

The title of this article, which no doubt impresses one peculiarly, refers to a certain type of textbook in use in secondary schools. The material in these books is usually presented in a psychological, rather than in the less interesting and logical manner. To many mature laymen, this represents an ideal way, the subject seemingly being easily mastered by a series of scarcely perceptible gradations. An important scientific term is scattered here and there, and at least until the student has progressed, say a half year, he is not troubled with those brazen-faced classifications which were the nightmare of earlier students in secondary science.

This type of book is just what the title implies—something which one, without some knowledge of botany and desiring some, can sit down and acquire without extraordinary exertion. Unless the above be construed as such, I have no criticism to offer upon this type of book, as I feel its non-justification may only be sought for after we have had more experience with the elementary science movement, when we may find that the high-school student's intellect is not as limited as sometimes we assume, and quite capable of acquiring logically a training in scientific method. The gist of the latter statement is, therefore, that our science courses must remain largely informational for a time, for which this type of book is seen to be well adapted. To the teacher whose enthusiasm is only too often killed by the "drill" process, it offers, as may be seen in the following, the satisfactory fulfillment of the old master's injunction, "Study Nature, not books."

I shall endeavor, as briefly as possible, to sketch the application as I have found it practicable, it being assumed that an ideally short and comprehensive method of keeping a laboratory notebook has been adopted. To begin with, no demonstration as described in the textbook is left merely described—it is already reduced to lowest terms. Right away up go hands in horror, "Where have we the time to do all of this?"—the work anticipated. Only to a limited extent does the ability to fulfill the program depend on the climate of environment surrounding the school. In the latter, if it has no greenhouse, may always be found some place where the few requisite plants will thrive. Often the teacher complicates the problem by insisting on a

tropical plant which may be indicated by the text, when a more homely and "almost as good" substitute grows near the school premises. Then with the formality which is now *passé,* the teacher refrains from requiring the students to collect commonplace material required, or even hesitates to accept it if offered from the pupil's home. Or, those thoroughly essential components of the botany course—field trips or visits to local conservatories, the year around—are neither taken nor utilized to collect material when so many willing aides may be found. Finally, odds and ends of time during periods of instruction—the technic once acquired—can be used.

To make the above fit in as it should, the teacher will make a comprehensive survey of his textbook, and endeavor to maintain a plastic, adaptable mind with regard to material the season may indicate. It will be remembered that part of the pupil's education is training in methods of study; consequently, when it is seen that such real application is necessary in certain parts of the book in order that the ground be covered, it should be strictly required. The teacher should be able to indicate, from his knowledge of the psychological processes, the manner in which the reading should be done at home. My own method for the latter is to take a few minutes a day for a month or two, in endeavoring to guide their train of thought along associated lines (for it will be remembered that the reading type of botanical textbook has not the skeleton of a college type), and where home reading is given, endeavor to point out by not too searching questions, the paths they should follow.

Laboratory work is most successfully given every day with wonderfully "sticking" results for the whole subject, as one may test for himself. If, for example, the types of parts of leaves are brought out in the subject of the day's lesson, they are shown the student and not always the same volunteer names them, or the teacher makes a rough sketch on the blackboard. Then the whole class with the material before them, draws and appropriately labels them. Or the description, as given in the textbook, may be read aloud jointly, or singly, with the teacher giving a synchronized demonstration.

The technic of presenting the matter of microscopic structures is more complicated. In my opinion, usually a great deal of microscopical work is far from beneficial as subtracting from the great whole. Yet we may continue our policy of seeing in a way that even the poorly equipped country high

school with its one or two decrepit microscopes can handle. For example, consider the structure of a leaf. Sections may be prepared with the hand sectioning razor, and successively seen by the members of the class, who, in the meantime may copy the illustration given in the textbook of the same object—the latter in itself a valuable experience. Many plants give all essential details even with the dissecting microscope, of which there are usually more in the schools.

The demonstrations of the physical and chemical phenomena associated with plant life are things which adolescents easily hold—especially if strikingly given—yet no complicated apparatus is necessary. As indicated above, the subject matter is read slowly or brokenly, as the demonstration proceeds, and the matter is wound up by the pupils making a free hand sketch of the apparatus. It is wise to let botany be the outlet for every other talent of the pupil which can be made to bring to bear upon it.

A happy simplification of the teacher's work can be found in the results from pursuits along these lines. High-school laboratories are often littered with a variety of things, usually well-meaning apparatus which has passed its day, or whose manipulation requires more time than can be given. Without endeavoring to furnish a list of required apparatus and supplies it may be said that it is astonishing how little of these are necessary to balance with what, at our present stage, we think the pupil should have, and still have him see the knowledge given in the reading textbook.

A few suggestions as to what I have found worth while. Leaves are everywhere—they may be dried and pressed for future needs. A hardy aquarium plant, easily obtainable at pet stores or conservatories is *Elodea,* excellent for the photosynthesis experiment, whose leaves may also be made to show cell structure and protoplasm. *Amaryllus,* a common household plant, is splendid for epidermal structures, cross section of leaf, while its subterranean parts when split lengthwise show admirably the structure of the root. Preserved, it may be used year in and out. Radish seedlings, sprouted on moist filter paper, in a Petri dish, are probably most convenient for root hairs. The screw pine, *Pandanus,* is excellent for stomata, which may be seen under the ordinary dissecting microscope. Lilac buds may be produced any time. Blocks of locust poles, four inches ong, cut in three planes, with a portion of bark still attached,

show conveniently a woody stem, while pieces of corn stalks, preserved or raised in the laboratory, are handy for monocotyls. The less experienced teacher does well to have on hand two or three laboratory manuals as provocative of ideas.

Unforseen interruptions like those of late tend to demoralize the school year, often cutting down the ground to be covered, but in seeing the botany course through, the great groups of plants and their subdivisions must not be forgotten, some definite knowledge must be given (not necessarily of reproductive phases below seed plants), and some definite mental picture must be formed as to what are, with their importance to life and relation to one another, the algae, fungi, lichens, bacteria, mosses, ferns, conifers, and palms, neglecting which high-school botany has scarce continued the knowledge previously imparted through Nature study and geography.

HORNER'S METHOD.

By Elizabeth Sanford,
Freeport High School, Freeport, Ill.

Have you ever asked why asylums were filled
With people whose sweet dispositions were killed?
It's because of one man, with a Mother Goose name,
Who invented a system to drive folks insane.
That's Horner.

Have you ever figured six sheets and a half
And sighed with relief that so much work was passed,
Then had the first figure point at you in glee,
Because you had put down a four for a three?
That's Horner.

Have you ever wished to commit suicide,
From this vale of numbers forever to glide,
When you've worked and you've cussed for an hour or more,
And the problem is not as correct as before?
That's Horner.

Have you ever gotten the answer at last,
Then turned to the page where last judgment is passed,
"Just to see if its right"—though there's no doubt at all—
And found you're as close as ping-pong to baseball?
That's Horner.

Have you ever wished that you just had the man
Who maliciously thought of that dastardly plan?
Hemp or bullets would be far too good for the beast,
We would have him work 10,000 problems at least.
That's Horner.

FORCE, WORK AND POWER—THEIR RELATION. NOTE ON RELATION OF WORK TO HEAT ADDED.

By S. A. Garlick,
Lyons Township High School, La Grange, Ill.

FOREWORD.

I have found the need of relating the study of Force, Work, and Power more closely one to the other than is done in the treatment accorded those subjects in the texts. In my work I have found it advantageous to show, in a concise way, that Work is evolved from Force, and Power is a concept enlarging upon the student's idea of Work.

The following article has been mimeographed and a copy handed to each student in the class. It is thought that this material might prove of help to others, and so is published in this magazine. Of course, a teacher could amplify and digress as he or she saw fit. For instance, force might lead to a comparison with pressure. Work might lead to the study of energy and momentum.

FORCE.

Force is defined (1) as push or pull, and (2) anything that produces motion or change of motion. Force may be measured in terms of two systems of units, (1) gravitational and (2) absolute. These two systems of units are necessary because the force of gravity varies at different places on the earth. For example:

G. at equator = 978.1
G. at poles = 983.1
G. at N. Y. City = 980.15

Before considering the units used in measuring Force any further, we must bear in mind the fact that a gm. and a gm. of Force are two different conceptions. A gm. stands for an amount of matter; a gm. of force for the pull of the earth on a mass of one gm. Obviously, as the pull of the earth varies slightly at different localities, this distinction is worthy of consideration when work of an exact and highly accurate character is being done. The gravitational units are not constant for they are based on the weight of a standard mass, but this weight might vary from place to place on the earth's surface. However, this system is satisfactory for everyday use and for the engineer.

In the absolute system a dyne is the unit of force and is defined as that force which imparts to one gm. mass an acceleration of one cm. per sec. per sec. This unit is invariable. In the gravitational system the gm. of force is the unit employed in measuring force and is the pull of the earth on a mass of one gm. definitely

at sea level and latitude 45 degrees. Change the locality and your gm. of force will change.

Gravity gives one gm. mass an acceleration of 980 cm. per sec. per sec. while a dyne, by definition, produces only one cm. per sec. per sec. acceleration. Therefore, one gm. of force equals 980 dynes at New York City, or 978.1 dynes at the equator.

Tabulation of units:

1 dyne = 1/980 = .00102 gm. of force at N. Y. City.
To change gms. of force into dynes, multiply by 980.
To change dynes of force into gms. divide by 980.
F equals $m \times a$ (see definition of units above).

WORK.

When a force moves a body it does work. To the physicist no work is ever done unless the force succeeds in producing motion. In short, in the scientific sense, work summarizes accomplishment, not effort.

$$W = f \times s, \text{ or } W = m \times a \times s.$$

Units of work are gravitational and absolute in character, just as was shown in our study of force. In the gravitational system it is worth while to consider units derived in terms of both the English and metric systems of measuring. The English unit is the foot-pound. In the metric system we often hear of a small unit, the gm. cm., and a large unit, the kg. m. The latter unit is 100,000 times larger than the gm. cm.

As the dyne is the unit of force in the absolute system, and as work is expressed as force times space, we may substitute dyne for force as one factor, and cm. for space as the other factor in the equation w equals $f \times s$. Thus the equation reads, w equals dynes times cms. When so expressed, a new unit is conceived and is called an erg. Thus the unit of work in the absolute system is called an erg, and is defined as the work done by a force of one dyne working through a distance of one cm.

$$e = d \times cm.$$

If gms. of force is unit given instead of dynes, and ergs are desired this equation would apply: e equals gms. x 980 x cms (See definition of gm. of force.) We can call an erg a dyne cm., if we so desire. But please note, we summarize the dyne cm. conception by using a shorter word, erg, just as we summarize two individuals, Mr. and Mrs., by using the term "couple." Further, it is interesting to note that no term has been used as a substitute for the expression foot-pound and gm. cm.

A nickel weighs nearly 5 gms. If we lift it from the floor to a table one meter high, we do 490,000 ergs of work. 5 x 980 x 100

equals 490,000 ergs. We know that in doing this, we would hardly think we had done much work, yet the figure 490,000 looms up and suggests that a great deal has been done. Supposing we expressed a considerable amount of work, such as lifting 50 pounds upon the table. As the figure expressing ergs of work done in this case would be enormous, and as such a figure would prove cumbersome in making any calculations in which we might want to use this figure, physicists have conceived of a much larger unit of work, called a joule. This unit is 10,000,000 times larger than an erg. 1 joule equals 10^7 ergs equals 10,000,000 ergs. The scientist has, in this case, taken a step such as coal dealers did when they invented the unit known as a ton for measuring coal—and for the same reason. Both want to work with as small numbers as possible and think in terms of large units. In lifting up the nickel, .049 joule of work was done. This is more in keeping with our idea of the little work involved.

One foot-pound work = about .738 or 3/4 joules.
One gm. cm. work = 980 ergs.
$f = ma$
$w = mas$

In defining force, we limit our consideration to two factors, mass and acceleration. In the case of work, we enlarge our conception to include space or distance. If we include another factor—"time"—in our conception, we have arrived at a definition of power and we have acquired another mental tool. The term power is commonly heard, but few people understand what they mean when they use this term. If in our definition of work we do not consider the element of time, it is obvious that time is no object. Thus, if we could get a man to volunteer to move some furniture for us we would not care whether he was all day about a two hours' job or not, but if we had to pay him by the hour we would immediately become interested in hi power or ability to work. The time factor would be of prime importance. In either case, the amount of work done would be the same but the rate of doing it would be different.

POWER.

Power is the time rate of doing work.
Power = work/time = $(f \times s)/t$
This equation may be expressed as (1) ft. lbs. per sec.
(2) gm. cms. per sec.
(3) ergs per sec.
(4) joules per sec.
Depending upon unit used in the numerator in expressing f or s.

Let us summarize the growth of our conceptions by tabulating them.

$$f = ma$$
$$w = mas$$
$$p = \frac{w}{t} = \frac{fs}{t} = \frac{mas}{t}$$

The units of power foot-pound sec., gm. cm. sec., etc., are not commonly used. We usually hear of the units of power called watts, kilowatts and horsepower. The expression horsepower is a condensed version of 33,000 ft. lbs. per min.or 550 ft. lbs. per sec. We might say that ft. lbs. per sec. had been submitted to a gathering up process, or, better, the assembly of diverse units into a usable form.

Thus, given f in lbs. of force, s in feet, t in sec., we arrive at this formula:

$$\text{H. P.} = \frac{f \times s}{550 \times t} \text{ or, if } t \text{ be taken as min., H. P.} = \frac{f \times s}{33{,}000 \times t}$$

In our everyday experiences, the terms watt and kilowatt are not usually used in speaking of the power of steam and gasoline engines. Horsepower is the expression almost exclusively used as the basis for rating the power of such machines. In the case of electrical machines, their power is always expressed in terms of watts and kilowatts.

The mathematical definition of a watt may be given in different ways, but for our present consideration we will confine our attention to the absolute or c. g. s. units. One watt is, of course, the rate of doing work and by definition is one joule per sec. Previously, we have seen that a foot-pound of work equals about 3/4 of a joule. Thus, a watt is a rate of doing work about 3/4 as rapid as a foot-pound per sec. A kilowatt is a unit, 1,000 times greater than a watt. One K. W. equals 1,000 watts. This summation is similar to the one found in linear measure, namely, that one meter equals 1,000 milimeters.

Let f equal dynes, s equal cms., t equal sec. in the following equation.

$$\text{Watts} = \frac{f \times s}{t \times 10^7} \qquad \text{Kilo-watts} = \frac{f \times s}{t \times 10^{10}}$$

1 H. P. equals 746 watts, or about 314 or .746 K. W.

To change K. W. to H. P. (approximately) add 1/3; to convert H. P. to K. W. subtract 1/4 to or from figure given. For example:

30 K. W. = 40 H. P.
50 H. P. = 35.5 K. W.

We often hear the term Kilowatt hour. This means a power

of one kilowatt acting for one hour and is useful in determining the charge which shall be made for electrical current service.
Thus, 10 K. W. hour equals 10 K. W. for 1 hr. or
1 K. W. for 10 hrs. or
5 K. W. for 2 hrs.

In the study of work, power and machines, losses due to friction and heat have been discovered. We cannot go very far in the science of physics without first finding out whether or not there is any connecting link between our units of mechanics and our heat unit. If we drive a nail, we do foot-pounds of work, and the temperature of both nail and hammer are raised, as we all know. Some of the work drove the nail into the board, and some of it was converted into heat.

And now the question is, how much work must be done to generate any given quantity of heat. The unit of heat with which we will concern ourselves is called a calorie, which is, by definition, the amount of heat which will raise one gm. of water 1°C. It has been found that 427 gm. meters of work produce 1 calorie of heat. This is equivalent to 42,000,000 ergs of work.

By means of this relation of work to heat we may begin by expressing ourselves in terms of one and convert our expressions finally into those of the other.

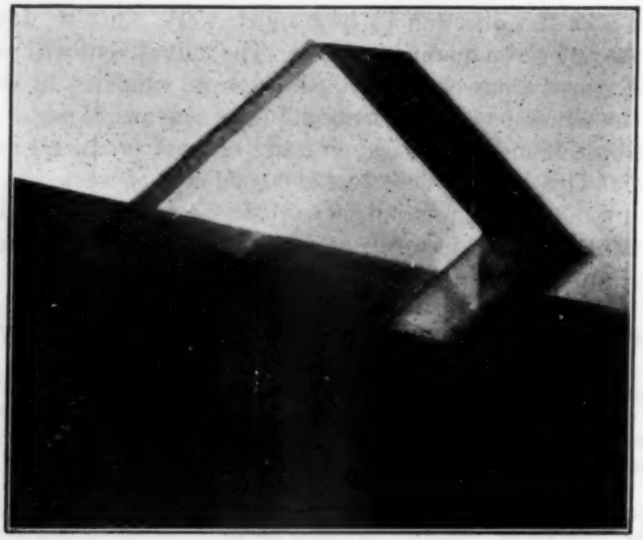
POSSIBLE OIL AND GAS IN EAST-CENTRAL NEW MEXICO.

In an examination of the geologic structure of parts of New Mexico made during the last four years by the United States Geological Survey, Department of the Interior, N. H. Darton, geologist, has discovered in the beds of rock many domes and arches that may be reservoirs of oil or gas if these substances are present in the regions examined. These flexed beds are parts of formations that produce oil in Wyoming, Oklahoma, Kansas, and Texas, but little evidence of the presence of oil and gas in them has yet been found in New Mexico. A small amount of oil at Dayton and a few seeps and some traces of oil reported in water wells at several other places are the only favorable indications so far reported. Only a few deep wells have been bored in New Mexico, however, and these have been bored in places where the structure was not favorable for the occurrence of oil or gas, or the wells have not been drilled deep enough to make them satisfactory tests. Much of the geologic guidance used in locating the wells drilled has come from incompetent "experts," one "dome" having been deduced from haphazard dips taken from layers of cross-bedded sandstone. One probably hopeless project is a deep hole in the middle of Tularosa Desert, where there are no rock outcrops to indicate structure. In some of the hectic literature written to promote the sale of oil stock the chief geologist of the United States Geological Survey is falsely quoted as authority for the statement that New Mexico would develop one of the biggest oil fields on this continent. There is no foundation for such a statement.—[United States Geological Survey.

A SHADOW METHOD OF MEASURING THE INDEX OF REFRACTION.

By A. A. Knowlton,
Reed College, Portland, Oregon.

Several of the elementary manuals describe a method of measuring the index of refraction of a glass plate by locating the image of a ruler or of a line established by two pins as seen through the plate. While the quantitative results are quite satisfactory this procedure leaves much to be desired on the qualitative side. The actual phenomena involved (the change in direction of the light rays at the bounding surfaces) is not seen directly but is deduced from the displacement of the image, and there is a total lack of what may be termed the scenery of an



experiment; i. e., coincident phenomena which are related to the measurement in hand in such a way as to attract the attention and arouse the curiosity of the observer. Moreover the manipulations involved, including the squinting through the glass and the exact placement of pins while in a cramped position, are of the fussy sort which most people find tiresome. All in all it is probably a fair example of the strictly quantitative laboratory experiment. The average student is likely to get little from it save the wholly uninteresting fact that the index of refraction of plate glass is about 1.52. Neither the connection between the measurement in hand and the phenomena of light which he sees in daily life nor the relation of this measurement

to the general laws of optics are brought into the foreground.

A modification of this method suggested by Mr. J. H. Frazier, a student in this laboratory, is free from the objections mentioned and at the same time retains all the advantages of the original method. It consists in observing the path of a shadow through the plate. One side of a rectangular glass plate having polished edges is painted with white enamel paint or, better, with the white ink used in libraries. This plate is then placed so that the shadow of any object with a straight edge falls across it as shown in the figure. Under these conditions one sees the undeviated shadow, the shadow produced by the light passing into the edge of the plate and both shadows beyond the plate as well as a number of other effects due to various reflections and refractions. The quantitative measurements are easily made with a satisfactory accuracy and the time consumed in a single measurement is so short that a series sufficient to really confirm the law of refraction may be made in half an hour. If one draws two perpendicular lines on a sheet of cross-section paper, describes a circle of ten centimeters radius about their intersection, places an edge of the plate on one of the lines and adjusts the object so that the shadow passes through the center of the circle the sines of the angles of incidence and refraction can be read off directly. It may be assumed with safety that the student who can give a clear account of the way in which the various shadows and spots of light are formed as well as tell why it is necessary to paint the glass instead of merely laying it on a sheet of white paper has a good working knowledge of the subjects of reflection and refraction. The shadow used should of course be as sharp and dense as possible. While satisfactory results have been obtained in a lighted room using a shadow cast by direct sunlight, a single distant window or an incandescent lamp, the results obtained in a darkened room and with a line source are of course much better. The best source which we have found for this as well as for other experiments requiring a line source is an ordinary nitrogen filled tungsten lamp (100 watt) with the horse shoe shaped filament. When the lamp is turned so as to bring the filament into a vertical position it affords a line source of great intensity.

THE BENT LEVER AS USED TO MEASURE THE MOMENTS OF PARALLEL FORCES.[1]

By Frank R. Pratt,
Rutgers College, New Brunswick, N. J.

The apparatus ordinarily used in teaching the moment of a force by laboratory experiment, is a meterstick supported at its center by a knife-edge and weights fastened to other knife-edges at certain distances from the center. The student forms the impression from this apparatus that the arm of the force is always equal to the length of the bar to which the force is attached. An experiment with the bent lever helps to correct this erroneous idea.

The bent lever is usually fastened in a horizontal position with its axis vertical to overcome the action of gravity and to keep the bent lever in equilibrium in all positions. The forces are applied in a horizontal direction by means of spring balances or by means of weights supported by cords which run over pulleys. There are several objections to this arrangement of apparatus. *First,* the spring balances do not read correctly when they lie in a horizontal position. *Second,* the pulleys have friction. *Third,* the arms of the force are not easily measured. *Fourth,* the apparatus occupies considerable space.

The following is a description of a bent lever apparatus which I designed to overcome the above objections. This apparatus helps the student to get the right impression of the arm of a force and can be used with profit in place of the meterstick apparatus. This bent lever is fastened to the cross-bar of the laboratory table by means of a clamp, A, Fig. 1, and all its parts lie in a vertical plane at the center of the table. This leaves the table free for other apparatus. The axle, B, is fastened to clamp A. The spider, C, which turns upon the axle, B, has ball bearings, one at each end, to overcome friction. The three arms of the spider, C, are arranged around the axle at an angle of 120° apart. The lever arms, D, are riveted to two of the arms of the spider, C. These arms are 35 centimeters long. Near the end of each lever arm is inserted a steel pin which supports the wire loops, J. The cords used to support the weights are fastened to the wire loops, J. Small rings are attached to the lower ends of the cords, and weights with hooks are hung on the rings. Weight, L, which is suspended from the center of the axle, B, acts as a plumb-bob. The zero of the scale, K, is set directly

[1]This piece of apparatus is manufactured and sold by the Standard Scientific Co., 70 Fifth Ave., New York.

behind the plumb-line. The scale, K, is supported by the vertical rods, I, and the clamps, H, which fasten to the cross-bar. A threaded arm, E, which supports the counter-weight, G, is riveted to the third arm of the spider, C.

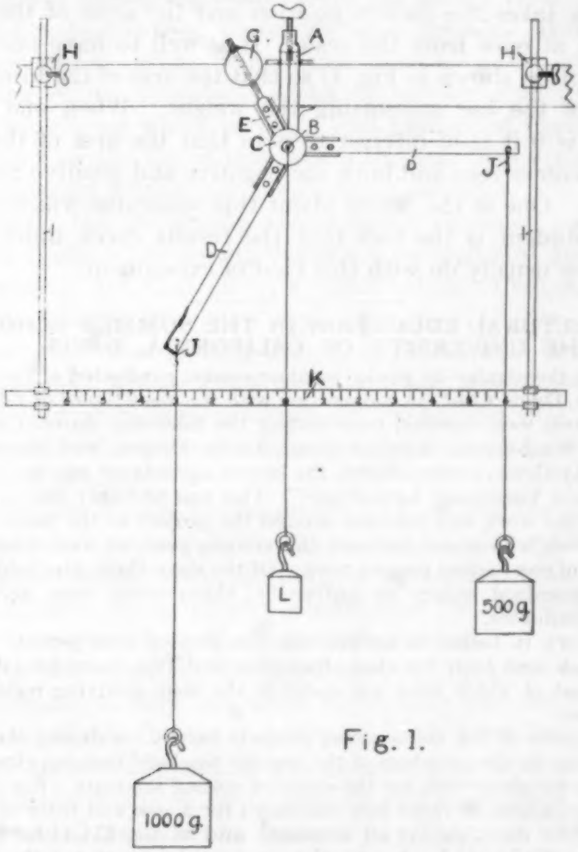


Fig. 1.

When the weights are removed the counter-balance weight, G, is screwed up or down until the center of gravity of the bent lever is at the center of the axis. This is determined by adjusting until the lever will remain at rest when turned in any position whatever. When the right position for the counter-weight is found a set nut fastens it securely and the apparatus remains in adjustment. The cords which support the weights hang in front of the scale and very close to it but they do not touch it. The scale shown is an arbitrary one used to show the principle. This scale should be divided at least as fine as millimeters. The estimation of tenths of these divisions aids in making the sum of the calculated moments equal to zero.

The student is not required to make careful adjustment of the position of the weights and add to these weights the mass of a knife-edge. The weights applied to the cords are the forces. The cords show the direction of the forces. The bent lever automatically takes the correct position and the arms of the forces are read at once from the scale. It is well to have one set of weights (that shown in Fig. 1) so that the arm of the force is the length of the bar supporting the weight. When one of the weights is 0 it is of interest to note that the arm of the other force becomes zero and both the negative and positive moments are zero. One of the points about this apparatus which appeals to the student is the fact that the results check much better than they usually do with this kind of experiment.

AGRICULTURAL EDUCATION IN THE SUMMER SCHOOL AT THE UNIVERSITY OF CALIFORNIA, DAVIS.

During the regular six weeks' summer session conducted at the University Farm, Davis, Calif., in cooperation with the State Board of Education, 112 students were enrolled representing the following states: California, Arizona, Washington, North Dakota, Idaho, Oregon, and Nevada. Of the twenty-three courses offered, the largest attendance was in "Methods of Teaching Vocational Agriculture." This was probably due to the reason that the work was centered around the project as the basis. Of the eighty-seven who expect to teach this coming year, all were interested in methods of conducting project work. Of the sixty-three who held degrees from a standard college or university, thirty-seven were agricultural college graduates.

The work in technical agriculture was divided into periods of three hours each, one hour for class discussion and two hours for laboratory work, most of which time was spent in the field, studying regular farm conditions.

The results of the thirty-seven projects carried on during the regular school year by the members of the regular teachers' training classes were available for class work for the summer session students. For example, one student demonstrated how she cared for a sow and litter of pigs for one hundred days, paying all expenses and having $51.44 for her labor income, or $0.78 per hour. Another student demonstrated the handling of 124 chickens during spring semester, making them pay their feed bill, rent, and depreciation, and $70.59 or $0.72 an hour for labor. The project work in the teacher training school at Davis last year included the following: Hogs, goats, poultry, vegetable gardening, products of various kinds, rabbits, and several varieties of fruit.

Every student made an investment and was held responsible for the conduct of the project. The profit or loss belonged to the student, since he took the risk. The projects were carried to a very successful conclusion. It is believed that if the teacher is to be successful in teaching by the project method in the high schools of the state, he must have had first-handed experience in conducting a commercial project and keeping an exact record of his business transactions. The new method of making the project the core around which instruction was given in the teacher training classes has been very successful.

S. H. DADISMAN.

A SENSITIVE GALVANOMETER FOR THERMO-ELECTRIC WORK.

By C. C. Kiplinger,

Mt. Union College, Alliance, Ohio.

The art of measuring temperature has been a valuable factor in the development of chemistry and physics. Beckmann's invention of the thermometer which bears his name marks the beginning of an important epoch in the history of physical chemistry, during which period a vast amount of experimental data based on the use of this instrument has been accumulated and augmented greatly our knowledge of solutions.

However, the Beckmann thermometer possesses some outstanding faults, chief among which are its high cost and fragile construction. Dr. Cottrell, in a recent article, "Journal of The American Chemical Society," Vol. 41, 5, p. 721, "On the Determination of the Boiling Points of Solutions," describes a "thermoscope" which seems to have some special merit, but which is not at present available on the market and the construction of which presents difficulties transcending the skill of the average worker.

Electrical resistance thermometers and thermopiles also have been employed successfully in the making of accurate temperature measurements. It is the purpose of this article to describe an installation set up in our laboratories for measuring small temperature changes by the thermopile method.

For the latter method, the usual form of apparatus requires a potentiometer, a very expensive instrument and hence, manifestly unavailable for the majority of small laboratories. The herewith described apparatus was designed with a view to eliminating the cost factor so far as possible and at the same time retaining some of the elements of accuracy. The availability of this device hinges on a galvanometer of improved design, by the use of which a higher degree of sensibility is attained than is possible with the older forms.

This instrument is of the astatic type, containing a single coil so shaped as to give the effect of several coils. One pound of No. 24, D. C. C. copper wire was wound on a wood shuttle having a core 1 1-2"x8"x3-4". The rectangular coil thus formed was then shaped by bending as shown in Figure 1. A round stick formed a useful adjunct in accomplishing this end. My students have facetiously called it "the pretzel coil." Five folds or loops permit the use of an astatic combination of six magnetized

needles. Evidently, it is feasible to use more loops and needles, should the occasion require. The magnets were attached to a very fine capillary glass tube by means of sealing wax and paper stirrups, giving a combination of minimum weight. A silvered bit of microscope cover glass was fastened to the glass tube with wax and constituted the mirror, permitting the instrument to be used with a telescope and scale. The needles and support were suspended by an unspun silk fibre.

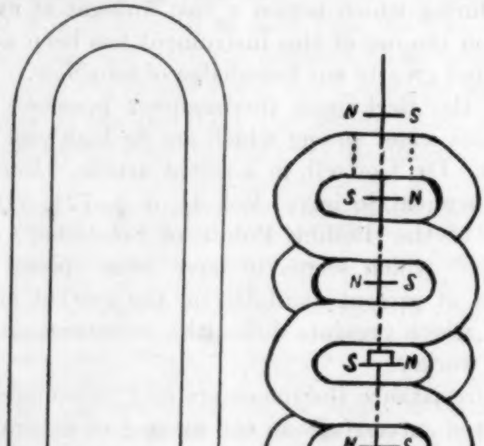


FIG. 1.

A pointed stick was forced carefully through the top of the shaped coil so as to make an opening through which the needles and support could be passed. The winding and shaping of the coil required less than two hours, giving a maximum sensibility for a minimum expenditure of time and effort in its construction. The coil was then soaked in paraffin and mounted as shown in the photographic reproduction, Figure 1.

An auxiliary coil containing about one hundred turns of wire, with a diameter equal to a single fold of the main coil, was so mounted on a graduated bar attached to the base of the instrument that its distance from the needle system could be varied and measured. This serves as a compensating coil and is con-

nected with a source of current independent of the galvanometer circuit. A telescope and scale mounted a meter and a half from the instrument completes the galvanometer equipment.

The thermo-element which has been in use with this galvanometer is composed of eight iron and german-silver couples in series. The german-silver wire is No. 24, D. C. C. and the iron bare wire, No. 30, used for standardization purposes in quantitative analysis. The couples are insulated from each other by passing them through channels in a silk ribbon, which channels are formed by folding a ribbon, five inches wide and as long as the couples, in half lengthwise then sewing parallel stitches lengthwise, spaced about 3-16″ apart. A couple is inserted in each channel, soldered to its neighbor, iron to german-silver, and the ribbon is then rolled into a small cylinder with the couples running lengthwise. This is mounted in glass tubes and paraffin as described by W. P. White, "Journal of the American Chemical Society," Vol. 36, No. 11, p. 2292. The german-silver wire was tested for homogeneity by White's method and was found to be satisfactory. The iron wire showed some inhomogeneity but not to a degree which would exclude its use for measurements accurate to at least .01°.

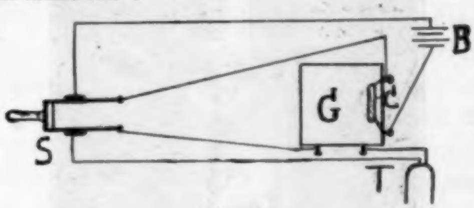


FIG. 2.

For the determination of molecular weights by the boiling point method, the electrical connections are made as shown in Figure 2. S is a small double knife switch of the usual type. Two cells of the constant current type are connected through C, the compensating coil, and one side of the switch, while the other side of the switch is a part of the galvanometer and thermopile circuit. This arrangement permits opening and closing both circuits simultaneously.

Each limb of the thermopile, T, is inserted in a boiling tube of the Jones or better, of the Cottrell type as described in the reference given above. The weights of these tubes should be known. A weighed amount of pure solvent is put in each tube and heated to the boiling point. We would expect the same

temperature in both tubes and consequently no current through G on closing S. However, this was never the case in practise, G always showing some current. By varying the distance of C from the needle system and reversing the direction of the current when necessary, the zero throw was minimized. Sensitive results were obtained by repeatedly opening and closing the switch at such time intervals as corresponded with the vibration period of the needle system. Very minute currents can be detected by this impulse method.

FIG. 3.

The compensating coil may be obviated by adding small portions of some solute of known molecular weight to the cooler tube until no current passes through the galvanometer. A known weight of the same solute is now added to this tube and the second solute, the molecular weight of which is to be determined, is added gradually, a small amount at a time, from a weighed portion, to the other boiling tube until thermal equilibrium has

been reestablished as indicated by the zero reading of the galvanometer.

The molecular weight of the second solute may be calculated from the above data by the use of the following formula:

$$M_u = \frac{W\ w'\ M}{W'\ w}$$

Where M_u = molecular weight of second solute—unknown;
M = molecular weight of known solute;
W = weight of solvent containing first solute—known;
W' = weight of solvent containing second solute—unknown;
w = weight of first solute;
w' = weight of second solute.

If each tube contains the same weight of solvent at the end of the experiment, then $M_u = \frac{w'}{w} M$, where M as above = molecular weight of known solute.

This procedure eliminates the boiling point constant C, the value of which always has been more or less uncertain. The method is also independent of changes in atmospheric pressure, since both boiling tubes are affected alike.

Before this investigation was entirely completed, an article, "Laws of Concentrated Solutions," Washburn and Read, appeared in the "Journal of the American Chemical Society," Vol. 41, 5, 729, in which this matter of the determination of molecular weights by the boiling point method is discussed at much length and in a far more efficient way than the writer is able to present it. However, the latter had shown the value of using a pure solute as a standard in such work in an earlier article, "Ebullioscopic Determinations with a Common Thermometer," "Journal of The American Chemical Society," Vol. 40, 7, 1020.

The experimental results obtained by the writer indicate a sensibility of the galvanometer and thermopile equivalent to .005°. If wound with smaller wire, this can be appreciably increased. It is hoped that this apparatus may prove of value to those who have to do with small temperature intervals and who have heretofore lacked proper equipment for this class of work.

HIGH VALUE FOR SEWER PIPE.

The third brick and tile product in value was sewer pipe. The output in 1918 was valued at $15,399,000. The growing popularity of this material in engineering works is shown by the fact that its decrease in value, $1,908,000, or eleven per cent, was proportionately small. The value in 1918 was the largest recorded except that in 1917, and, making a liberal allowance for increase in price the product in 1918, must have been larger than in any year prior to 1912. Considerable quantities of vitrified sewer pipe were used in Government projects at army camps and cantonments.

TESTING RESULTS IN CHEMISTRY.

By B. J. Rivett,
Northwestern High School, Detroit, Mich.

The teacher of Chemistry, if he does his work properly, is a busy man. In addition to his teaching he has to purchase supplies for his laboratory, correct note books, and do the thousand and one things necessary to keep a laboratory in good condition. Again, in many schools he teaches Physics and perhaps other subjects. It would be interesting to see the figures for the percentage of Chemistry teachers who coach or manage athletic teams.

The manner of conducting the recitation in high school Chemistry presents a problem different from most subjects. There are only three recitations a week on account of the two double periods required for laboratory work. In the recitation the teacher must develop new work, give demonstration experiments, and conduct an oral quiz over the work covered in the text and the laboratory. With large classes of twenty-five to thirty pupils it is impossible to question every pupil each day and sometimes pupils go a whole week without reciting. This method results in slipshod work on the part of pupils for they will not study unless they are quizzed frequently.

Until recently, to overcome this difficulty I have given a written test every two weeks, graded the papers myself, and returned the papers to the pupils. But this involves a large amount of work on the part of the teacher, averaging four hours every two weeks. During the past year I have devised a method for testing pupils which may prove to be of interest to other teachers.

Every week I prepare a test on the previous week's work which may be written in three to eight minutes; depending upon the nature of the work. These questions are mimeographed so that each pupil has a copy. The test is conducted in the same manner as a Courtis Arithmetic test, that is to say, there is a time limit on each question. When the time is up, the pupils exchange papers and the correct answers to each question are given. The pupils are told what per cent to mark for each correct answer. When the papers are graded they are returned to their owners for final inspection and then taken up by the teacher. This type of a test may be conducted in fifteen to thirty minutes and the pupils know exactly their knowledge of the week's work.

Below are given two types of tests which illustrate the method described.

TEST NUMBER 1.

CHEMISTRY (1)

1. Calculate the percentage of water in $CuSO_4, 5H_2O$. Cu equals 64; S equals 32; O equals 16; H equals 1.
Parts by weight: $CuSO_4, 5H_2O$.
Calculations:

2. Calculate the per cent of each element in HgO. Hg equals 200; O equals 16.
% of Hg is
% of O is

3. Name these: SO_3________ ________;

SO_2________ ________; Na Cl________ ________;

K Br________ ________; H_2 S________ ________;

Ca C_2________ ________; Na I________ ________.

Mg O________ ________;

4. (a) State the relation between the volume and pressure of a gas.

The volume of a gas varies ________ with the pressure, the ________ remaining constant.

(b) State the relation between the volume and the temperature of a gas.

The volume of a gas varies ________ with the ________________,

the ________ remaining constant.

5. A simple barometer is made by ________ ________ ________

________ ________ ________ ________ ________ ________

________ ________ ________ ________.

A barometer is used for ________ ________ ________ and for

________ ________ ________.

6. Nitrogen has the following properties:

(a) It ________ ________ ________.

(b) It ________ ________ ________ ________.

(c) It is ________________.

7. Absolute zero is ________ ________.

It is so called because ________ ________ ________ ________

________________.

Centigrade readings are changed to absolute readings by ________

________ ________ ________ ________.

TEST NUMBER 2.

Name ____________________

Time ________ min.

Indicate by a Roman numeral to the right and above the valence of the following elements and radicals:

Al	NO_3	Mg
NH_4	SO_4	K
Ca	PO_4	Ag
H	OH	Na
(2) Cu	Cu	Zn
(2) Fe	Fe	Br
(2) Hg	Hg	Cl

Write the formulas for the following compounds:

Sodium chloride ————	**Ammonium sulphate** ————
Aluminum chloride ————	**Calcium sulphate** ————
Calcium chloride ————	**Potassium sulphate** ————
Silver chloride ————	**Magnesium sulphate** ————
Zinc chloride ————	**Sodium sulphate** ————
Silver nitrate ————	**Sodium hydroxide** ————
Sodium nitrate ————	**Ammonium hydroxide** ————
Ammonium nitrate ————	**Calcium hydroxide** ————
Potassium bromide ————	**Aluminum hydroxide** ————
Calcium Phosphate ————	**Ferric hydroxide** ————

Some teachers may object to this type of test on the ground that pupils will not mark the papers correctly. I have tested this point and know that pupils can be trained to mark as accurately as the teacher. Some may argue that this test is a device and as such has only a limited use. It can be used repeatedly and if a device, it is a valuable one. True it does not entirely displace examinations for they should be given once or twice a semester. Again, some may believe that the time test gives no measure of reasoning power. This depends entirely on the nature of the questions asked; the questions need not be all of the memory type.

I wish to submit the following advantages for this type of test as compared to the old form of written test or examination:

1. There is less opportunity for some pupils to get help from others. The reason for this lies in the fact that there is no time for pupils to observe what others are doing; they must be busy every second to finish.

2. The time limit teaches the pupils to concentrate their minds. Pupils can solve a problem in two minutes. Why give them five or ten minutes?

3. The new method saves the time of the teacher. The teacher is relieved of the monotonous work of correcting papers.

4. The pupils know exactly how much of the work they have mastered to date. The teacher can judge how well he has done his work with this test as well as with a long examination.

Every high school should be supplied with a mimeograph and a pupil from the commercial department can make the copies of the necessary questions. Furthermore, there is no reason why the time test as conducted in Chemistry may not be used by teachers of other subjects. In conclusion, I wish to recommend the short time test to all busy teachers.

TWO LAWS TO CONTROL CATS.

By Horace Gunthorp,
Washburn College, Topeka, Kans.

A good many people object to the wholesale killing of cats in order to control the worst enemy our birds have, claiming that many pets and animals of value will be destroyed along with the many superfluous and worthless felines. It is true that there are always two sides to every question, and while the writer has very little sympathy to spare for either the cat or its fond owner, he does not care to trample on the rights or feelings of other people provided he can get a square deal for the birds without doing it. He believes that a fair solution of this troublesome problem can be reached by the enactment of two laws, the first of which has recently been passed in New York State, and is well summed up and its effects discussed in the following quotation, taken from a recent number of *The Wilson Bulletin*.

"Cats with a fondness for birds are in danger, for Governor Whitman has signed a bill providing for their destruction. 'Any person over the age of twenty-one years,' reads the law, 'who is a holder of a valid hunting and trapping license, may, and it shall be the duty of a game protector or other peace officer, to humanely destroy a cat at large found hunting or killing any bird protected by law or with a dead bird of any species protected by law in its possession; and no action for damages shall be maintained for such killing.'

"Cat bills of many varieties have, in the past, been presented to different state legislatures. Some have called for bells on cats, some for collars and licenses, but the present law is the first to be passed in any state. Its promoters have framed it with the idea of attacking, not the well-fed and cared-for house pet, but

[1]Vol. 30, p. 118. Dec. 1918.

the wandering, hunting, or homeless cat, which has become so great a menace to our wild-bird life.

"Figures gathered by the Conservation Commission indicate that common cats cause more destruction among insectivorous and game birds than any other agency. The present law is intended to encourage all persons, who are sufficiently responsible to carry a gun, to aid in checking the numbers of bird-hunting and bird-killing cats. The new law goes into effect immediately."

The above law is a good one, and is a long step in the right direction, but even its enforcement would leave a large number of worthless, half-starved, night prowling cats at large unless they individually happened to be caught hunting protected birds. Of course they would stand a good chance of being so taken, sooner or later, but in the meantime they could do a considerable amount of damage to our bird population. To meet this situation, let us have a second law (or ordinance) enacted by the municipalities which shall provide for the licensing of cats just the same as we now have laws so providing for dogs. This is not a new idea by any means, but it seems an appropriate time to again call attention to them at ter in connection with such a good law as that just passed in New York. The license fee should amount to at least one dollar for each male cat and three dollars for each female, and the money so collected would provide the necessary funds for the license tags, the salary of the "cat catcher," and any other expenses connected with the enforcement of the act. This ordinance, like the above mentioned state law, would be aimed at the same class of undesirable cats, although it is admitted it would place a great many more in such a class (where they ought to be), because as soon as the possessor found it was going to cost something to keep a cat, many of them would be sacrificed, and a small fraction of the license fee would be expended for a mouse trap or two to do the work of the cat. In fact, only cats that were held in considerable esteem either as pets, or because of real worth, would be able to survive the acid test of a yearly drain on the possessor's pocket book, and the homeless cats would simply become non-existent so far as places having this ordinance were concerned.

The combined result of the enactment and enforcement of the above two laws should meet the approval of not only all bird-lovers, but of the most strenuous defenders of the cat. The birds would be much better protected from their arch enemies, and the tragedies we so hate to witness and hear about would

certainly become less prevalent. On the other hand, the suffering of thousands of neglected and homeless cats would be prevented, and they would at once attain to a legal status and value never enjoyed before, two conditions all cat-lovers should welcome. Dealers in cats would certainly endorse such measures, as the demand for a higher class of cats, and hence more expensive ones, would increase as the common breed became less numerous. If a person has to pay a tax on an animal, he wants one that is worth paying for while he is about it.

Regarding the seriousness of the cat question, note above what the New York State Conservation Commission has to say. Any person who is familiar with the case will agree with Frank M. Chapman who thus sums up the situation, "The most important problem confronting bird protectors today is the devising of proper means for the disposition of the surplus cat population of this country." This being the case, we should enter upon a more strenuous campaign against this animal, and what group of people is better situated to bring about results than the teachers? Realizing that cat owners and cat lovers have rights, is it not possible for both sides to agree on a program similar to that outlined above, and then see that it is put into effect? There is only one way to get results in this world—that is to start something, and then push it along, in season and out of season. And now is the time to begin. The longer we delay, the more cats there will be, and more cats means fewer birds, and fewer birds means more insects, and more insects means less food and higher prices!

OPTICAL AND PHOTOGRAPHIC METHODS FOR THE DETECTION OF INVISIBLE WRITING.

Optical and photographic methods for the examination of suspected papers and documents, developed by the authors and used by the Office of Naval Intelligence and Military Intelligence during the war, included the following: (1) Examination by intense illumination at grazing incidence, (2) examination by regular reflection, (3) examination by fluorescence, (4) ultra-violet photography.

Special apparatus was designed and constructed at the Bureau of Standards to facilitate rapid and convenient routine examination by these methods. This is to be exhibited at the Physical Society meeting, April 25-26. Four confidential technical reports on this subject were made to the Office of Naval Intelligence, February to May, 1918, and a later report (August) by one of the present authors (Tyndall) in conjunction with Dr. N. E. Dorsey on the use of the electroscope and X-ray photography in detecting secret writing. A more detailed account of this work will appear in the forthcoming military reports of the Bureau of Standards.—*Physical Review.*

ARTICLES IN CURRENT PERIODICALS.

American Journal of Botany, for July; *Brooklyn Botanic Garden, $5.00 per year, 60 cents a copy:* "The Structure of Protoplasm," R. A. Harper; "George Francis Atkinson," W. G. Farlow, Roland Thaxter, and L. H. Bailey; "Publications of George Francis Atkinson," Harry M. Fitzpatrick; "Viability of Detached Root-Cap Cells," L. Knudson.

American Mathematical Monthly, for September; *27 King St., Oberlin, Ohio; $3.00 per year, 35 cents a copy:* "The National Committee on Mathematical Requirements;" "A Theory and Generalization of the Circular and Hyperbolic Functions," A. F. Frumveller; "A Proof of a Theorem of Compound Probabilities," Ugo Broggi; "Bits of History about Two Common Mathematical Terms," G. A. Miller;" Questions and Discussions," Questions by Harris Hancock; Discussions—"The Complex Quantity in Algebra," by W. W. Johnson, and "On Teaching of Logarithms," by R. B. McClenon.

Condor, for May—June; *Hollywood, Calif.; $2.00 per year, 40 cents a copy:* "Some Notes on the Egg of *Aepyornis Maximus*(with four photos), Wm. C. Bradbury; "Autobiographical Notes" (with portrait), Henry W. Henshaw; "A Return to the Dakota Lake Region (continued), "Florence M. Bailey; "Malcolm Playfair Anderson" (with portrait), Melville B. Anderson; "Description of an Interesting New Junco from Lower California," Harry C. Oberholser.

Journal of Geography, for September; *Broadway at 156th St., New York City; $1.00 per year, 15 cents a copy:* "Announcement of Prize Essays and the Policy of the Journal of Geography," by the Editors; "The New Boundaries of Germany;" "Outline of a Study of Europe from the Point of View of the War," J. W. Hubbard; "Organization of an Ideal Course in Geography," H. W. Fairbanks; "Flying Fish," Mark Jefferson.

Literary Digest, for September 13; *New York City; $4.00 year, 10 cents a copy:* "The Cummins Cure for our Railroad Distress; ""Hoover vs. Hapsburg;" "Our Warning to Turkey;" for September 27, "The Policeman's Right to Strike;" "Misgivings Regarding the Austrian Treaty;" "England's 'Iron Hand' on Ireland."

National Geographic Magazine, for October; *Washington, D. C.; $2.50 per year:* "A Vanishing People of the South Seas" (23 illustrations), John W. Church; "A Mexican Land of Canaan" (17 illustrations), Frederick Simpich; "Wild Ducks as Winter Guests in a City Park" (11 illustrations), Joseph Dixon; "Curious and Characteristic Customs of Central African Tribes" (35 illustrations), E. Torday.

Physical Review, for September; *Ithaca, N. Y.; $6.00 per year, 60 cents a copy:* "Studies of the Absorption of Gases by Charcoal. I. Variations Due to Heat Treatment, Pre-equilibrium Effects," Harvey B. Lemon; "Fluorescence and Absorption of the Uranyl Sulphates," E. L. Nichols and H. L. Howes; "A Critical Thermodynamic Discussion of the Volta, Thermo-Electric and Thermionic Effects," P. W. Bridgman; "The Magnetic Properties of Some Rare Earth Oxides at Low Temperatures," E. H. Williams; "Displacement from the Apparent Vertical in Free Fall," Will C. Baker; "A Note on the Comparison of Inductances, or of an Inductance and a Capacity by an Electrometer Method," Alva W. Smith; "The Most Probable Value of the Planck Constant h," Raymond T. Birge; "On the Critical Absorption and Characteristic Emission X-ray Frequencies," William Duane and Kang-Fuh Hu; "Propagation of Sound in an Irregular Atmosphere," G. W. Stewart.

Popular Astronomy, for August-September; *Northfield, Minn.; $3.50 per year:* "The Selection of Sites for Astronomical Observatories, (with Plate XXVII), William H. Pickering; "The Changing of the Date at the 180th Meridian," Albert S. Flint; "Leap Years with Fifty-three Sundays," Carl Boecklen; "A Year of Comets" (with Plates XXVIII-XXXIV), Maynard Shipley; "The Pulfrich Sextant," F. J. B. Cordeiro; "Southern Celestial Objects for Small Telescopes," Bernard Thomas; "Respecting a Possible Destruction of Human Life," Charles Nevers Holmes; "A New Transit Reduction Computing Machine," Edward C. Phillips, S. J.; "The Annular Eclipse of the Sun of 1919, November 22, as Visible in the United States," William F. Rigge.

Popular Science Monthly, for October; *New York City; $2.00 per year, 20 cents a copy:* "Fighting Fire from the Sky;" "After a Cocoanut Leaves the Tree," "What of Tomorrow's Flying?" "How that Carpet of Yours Was Made?" "Can You Make a Crate that Won't Break?"

Review of Reviews, for September; *New York City; $4.00 per year, 35 cents a copy;* "Hungary, the Balkans and the League;" "High Prices and a Remedy;" "The Plumb Plan and the Railways;" "Mexico: the Unsolved Problem."

Science, for September 12; *Garrison-on-Hudson, N. Y.; $5.00 per year, 15 cents a copy:* "Not Ten but Twelve," William B. Smith; for September 19, "A Basis for Reconstructing Botanical Education," C. S. Gages; for September 26, "The New Opportunity in Science," R. A. Millikan; "Chemistry in the Navy," Admiral Ralph Earle; for October 3, Epidemiology and Recent Epidemics," Dr. Simon Flexner; "The New International Union of Pure and Applied Chemistry," Edward W. Washburn.

School Review, for October; *University of Chicago Press; $1.50 per year, 20 cents a copy:* "Space-Provisions in the Floor-Plans of Modern High-School Buildings," Leonard V. Koos; "A Committee on Results," E. R. Breslich; "Geometry by Analysis," H. O. Barnes.

Scientific Monthly, for October; *Garrison, N. Y.; $3.00 per year, 30 cents a copy:* "The Origins of Civilization," James H. Breasted; "Man and His Nervous System in the War," F. H. Pike; "Applied Nutrition for Raising the Standard of Child Vitality," I. N. Kugelmass; "Cooperation and Individualism in Scientific Investigation," Dr. C. L. Shear; "In Regard to Species and Sponges," H. V. Wilson; "Ignis Fatuus," Fernando Sanford; "Linkages," Frank V. Morley.

INVESTMENTS FOR TEACHERS.

What investments are you making these days? Reconstruction in education promises to be a mighty interesting field for investigation. Our teachers showed their loyalty and worth during the war. The public at large is coming to a realization of the importance of good teachers, and salaries are on the up-grade. The opportunities ahead will be much greater. The teacher who is ready, who knows his ground, who is resourceful, will have a greater chance to prove his powers of leadership. Are you ready for these increased opportunities? Have you invested as heavily in educational preparation and educational associations as you should?

The Central Association of Science and Mathematics Teachers is a group of over 1,200 of the most progressive teachers of science and mathematics in the Middle West, including teachers in colleges, normal schools, high schools, and Junior high schools. They meet this year in the Lake View High School, Chicago, Friday and Saturday, November 28 and 29.

G. Stanley Hall is one of the speakers. He comes from the East to help in the work of the Association. Committees are now working on a REORGANIZATION PROGRAM in each section of the Association. Their reports will undoubtedly be of much interest and value.

The annual dues, $2.50, include subscription to SCHOOL SCIENCE AND MATHEMATICS. This is an investment in educational efficiency that we believe will pay you excellent dividends.

Plan to be present at the November meeting.

PROPOSED AMENDMENT TO THE CONSTITUTION OF CENTRAL ASSOCIATION.

The cost of printing has advanced at least fifty per cent during the past five years. As a result, the Association found it necessary to increase the rate of its subscriptions to SCHOOL SCIENCE AND MATHEMATICS.

This item alone has amounted to a twenty-five per cent decrease in the funds available for expenditure on current educational problems.

The Association regularly secures convention speakers of national reputation to discuss the new educational problems. Everyone is familar with the increase of railroad and hotel rates. These and many minor items have vastly increased the cost of providing the proper facilities for the advancement of science and mathematics teaching.

Last year it was voted to suspend publication of the *Proceedings* in the hope that an increase of dues would not be necessary, but in spite of this economy, the Association finds that it must have more income or further decrease the quality and the amount of its service to its membership. It is for the purpose of avoiding such a calamity that a committee, appointed from the Executive Committee, after thorough investigation, makes the following recommendation for an amendment to the Constitution, to go into effect, December 1, 1919:

Resolved that Article V be amended to read as follows: "The annual dues of active and associate members shall be three dollars ($3.00), payable at the annual meeting for the following year. Members in arrears for one year shall be dropped from the list of membership."

PEAT OUTLOOK IS ENCOURAGING.

The notable increase in the output of peat in recent years, especially in 1917 and 1918, is ascribed to the shortage and the high prices of nitrates and coal, to a nation-wide demand for increased food production that necessitated intensive agriculture, to a better understanding of the nature of peat, and to the application of bacteriology to crop cultivation. Though some of these causes of the increase in output were due to the war, all of them will probably continue to affect the industry in this country, so that the demand for peat products will correspondingly increase and peat will maintain or even better the position it took in 1918 in the industrial activity and progress of the United States.

A PLANET WHICH DID NOT EXIST.

As we all know very well, little Mercury is the nearest of the eight planets to our Sun. Possibly there may be some tiny planetary body between Mercury and the Sun, but that is improbable. Nevertheless, at one time, it was believed that such a planetary body had been discovered, for, on March 26, 1859, Dr. Lescarbault, at Orgeres in France, announced that he had beheld a small planet cross the Sun's surface, between Mercury and the Sun. The famous astronomer Leverrier visited Dr. Lescarbault, and was convinced that the Doctor had not been deceived. This new addition to the eight other planets of our Solar system was named Vulcan, and although other astronomers believed afterwards that they saw Vulcan, it is certain today that this alleged planet never existed. Not only was it given a name, but also its distance from the Sun was estimated to be 13,000,000 miles and its diameter 2,500 miles. Since Dr. Lescarbault's "discovery" of Vulcan, about sixty years have passed, telescopes and telescopic cameras have been vastly improved, but no planet between Mercury and our Sun has been observed. That tiny particles of matter exist on orbits between our Sun and Mercury, astronomers do not doubt, indeed larger particles may exist there; but after all the long and careful search of our Sun's surface and environments, with modern telescopes and methods, it seems impossible that any body deserving the name of planet is revolving between the Sun and the orbit of Mercury.—*Popular Astronomy.*

Nineteenth Meeting
of the
Central Association of Science and Mathematics Teachers

The Association meets this year at Lake View High School, Chicago, November 28 and 29. The Executive Committee is all set for a great meeting. Lake View has gone the limit in preparation, and all old and new members attending this year's meeting will certainly get the glad hand.

We are in a period in which reorganization is the word. Ideas and ideals in education are undergoing a distinct metamorphosis. Many of the standards by which administrators and the public measured the success of our courses in science and mathematics which held yesterday are in the discard today. It certainly will not be to our credit to take an apathetic attitude toward the job which is ours; we should not settle back and do nothing. We should exert our influence in determining the direction which reorganization takes.

The H. C. of L. demands economy. Economy demands efficiency. In these times we cannot afford to deny ourselves the mental tonic and professional uplift to be gained by an exchange of ideas with people whose problems have been identical with our own. The Central Association was developed to supply this need. It is growing because it is doing that service. "Watch the Central Association grow," and help it along.

Dr. Lynn H. Hough, President of Northwestern University, will speak on Friday morning on the theme, "Science and Humanism." Dr. Hough has given much attention to this vital topic and he is certain to inspire in us a new regard for our subject. The general theme for the convention this year is, "The Relation of Vocational Education and Vocational Guidance to Our Teaching of Science and Mathematics." Dr. Hough's address will involve the former phase of the subject. The latter phase of the subject will be discussed by Mrs. Anna Y. Reed, Assistant Director, Junior Division United States Employment Service, Washington, D. C.

The section programs, Biology, Chemistry, Earth Science, General Science, Home Economics, Mathematics, and Physics are particularly strong in having speakers of prominence who know secondary school problems. Space does not permit individual mention. However, a few topics selected from the programs will serve to emphasize the value of the Association to ambitious teachers. O. H. Benson, Specialist of the United States Department of Agriculture, will speak on the subject, "Projects in Biology and Agriculture." "Chemical Warfare and Chemical Teaching" is the subject of an address by Professor William McPherson, Ohio State University. Professor A. W. Nolan of the University of Illinois will discuss, "The Relation of General Science to the Smith-Hughes Courses." Rosa Biery of the University of Chicago Elementary and High Schools will read a paper on the very practical subject, "Economics in the Home Economics Course." "Commercial Geography as Vocational Guidance," will be presented by Andrew Nichols, Austin High School, Chicago. J. A. Foberg, Secretary of the National Committee on Mathematical Requirements, will discuss, "The Evans Report on a First Year Course in Algebra and Geometry." "The Automobile in Physics," is the subject of a paper by H. Clyde Krenerick, North Division High School, Milwaukee, Wisconsin.

Programs will be mailed about November 5. If one fails to reach you, send your name and address to the President, Jerome Isenbarger, Senn High School, Chicago, Ill.

SCIENCE QUESTIONS.

Conducted by Franklin T. Jones.

The Warner & Swasey Company, Cleveland, Ohio.

Readers are invited to propose questions for solution—scientific or pedagogical—and to answer questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, 10109 Wilbur Ave., S. E., Cleveland, Ohio.

Please send examination papers on any subject or from any source to the Editor of this department. *School examinations of all sorts are wanted. If you have run across anything queer, be sure to send it in.*

Think Questions.

What have you in the way of questions to make a boy or girl *think?* Send them in. Most questions are *information* questions rather than questions which promote thought. Any questions of this sort will be given space and credit in this department.

Acknowledgment.

The receipt of questions is acknowledged from Brother Charles Aul, St. Mary's College, San Antonio, Tex. Next!

QUESTIONS AND PROBLEMS FOR SOLUTION.

330. *Proposed by Worcester R. Warner to the boys in the Warner & Swasey Apprentice School.*

A room is 30 feet long, 12 feet wide and 12 feet high. A spider starts from a point in the middle of the end wall of the room one foot from the floor and crawls to a fly in the middle of the opposite wall one foot from the ceiling. How far does he travel if he goes by the shortest path?

331. *Proposed by William Becka, Instructor in The W. & S. Co. Apprentice School.*

A bullet is fired straight upward from a Colt automatic. The bullet has a rounded point with copper cap and a lead back. Which end of such a bullet comes down first? Why?

Which end strikes first if fired obliquely upward?

332. *Proposed by H. W. Corzine, Cleveland, Ohio.*

A bullet is fired parallel to the earth from a machine gun on an aeroplane at an altitude of 30,000 feet, with a muzzle velocity of 2,700 feet per second. At the moment of firing another bullet is dropped from the hand. Which reaches the ground first and by how much? Why?

Examination Lists.

Submitted by P. M. Dysart, Schenley High School, Pittsburgh, Pa.

MECHANICS.

First Test.

1. (a) Metric table of surface, giving the relation of each unit to the square meter.

(b) What is the mass of 1,000,000 cubic millimeters of pure water at 4°C?

2. (*a*) The mass of a body at the earth's surface is 10 kilograms, what is its weight at the earth's centroid?

(*b*) A non-homogeneous disc properly mounted upon an axis.

3. How long will be required to stop a car of 20,000,000 g. mass vel. being 1,200 cm.sec. and the brakes exerting an unbalanced force of 200,000,000 dynes?

4. (*a*) How great an unbalanced force is required to change the vel. of a 500 g. mass from 40 cm. sec. N to 60 cm. sec. N. in 5 seconds?

(*b*) A locomotive exerts a force of 55,000 lb. upon a 500-ton train. Friction amounts to 5,000 lb. Find the rate of acceleration of the train.

5. Using P to denote the vertical position of a pendulum, A, the extreme left-hand position, C, the extreme right-hand position, state:

(*a*) Position at which the unbalanced component of the weight of the bob is greatest.

(b) The rate of acceleration least.
(c) The velocity greatest.
(d) The rate of acceleration zero.
(e) The velocity zero.

6. (a) A ball is dropped from a car while the car has a horizontal vel. of 15 ft. sec. While the ball is dropping, the vel. of the car is negatively accelerated. Sketch the path of the ball with reference to the earth. Give the horizontal vel. of the ball.

(b) State the principles illustrated by the experiments with the block and stick.

7. (a) The mass of a gun is 40 tons, of the projectile, 800 lb. The velocity of recoil of the gun is 27 ft. sec. What is the vel. of the projectile?

(b) Find the distance in feet traversed by a freely-falling body in 5-8 second, initial vel. being zero.

8. (a) A body is projected vertically downward with an initial vel. of 15 meters per second. Find the distance traversed in 4 seconds.

(b) While swinging through an arc of 10 cm., the period of a pendulum is 2 seconds. What does its period become while swinging through an arc of 5 cm.?

9. (a) State two methods of lengthening the period of a balance wheel.

(b) A bullet has a horizontal vel. of 700 meters per sec. at the instant that it leaves the gun. How far does it travel horizontally while dropping 20 meters?

10. (a) Horizontal section of a centrifugal wringer. Show the path that the water would take, were there no gravity.

(b) Which is the more difficult to constrain to move in a circular path, a 10 g. mass, vel. 200 cm. sec., radius of path 100 cm., or a 20 g. mass, vel. 200 cm. sec., radius of path 200 cm.?

Submitted by P. C. Hyde, Newark Academy.

PHYSICS IV—JUNE, 1919.

Answer Ten Questions Only.

1. A wooden cube 6 cm. on an edge weighs 142 grams. (a) What is the density of the wood? (b) What force would be required to hold the cube submerged in a liquid of density 1.3? (c) State the principle on which your answer to (b) is based.

2. Draw a diagram of a force pump. Explain the action of the pump, referring as much as possible to your diagram.

3. What is the length of a pipe, closed at one end, which at 17 C. gives greatest reinforcement to sound produced by a tuning fork of 240 vibrations per second?

4. What are the essential parts of a storage cell? What are the chief merits of this cell that are not possessed by the ordinary cell? Name three practical uses of this cell.

5. Describe the construction and operation of two of the following: electric bell, telegraph key and sounder, telephone receiver. Illustrate by carefully drawn diagrams.

6. State the law of the conservation of energy. Explain how change of energy of one sort into energy of another sort is illustrated when an alarm clock rings.

7. A box weighing 150 pounds is pushed up a plank 10 ft. long into a wagon 3 ft. high. (a) What work is done against gravity in pushing the box from the ground into the wagon? (b) If a force of 100 lbs. is required to do this, how much work is done against gravity? (c) What is the efficiency of the arrangement?

8. A balloon is filled on a cool night with 20,000 cu. ft. of gas at a temperature of 7°C, under a pressure of 15 lbs. per sq. in. In the sunshine of the day the gas becomes warmed. Assuming that the bag does not stretch and that no gas escapes, at what temperature will the pressure become 16 lbs. per sq. in.?

9. Explain clearly the distinction between quantity of heat and temperature. State a method of measuring temperature and the facts

on which it is based. State a method of measuring quantity of heat, and give the principle involved.

10. Describe an experimental method of determining the index of refraction of glass. If this is found to be 1.5, what is the speed of light in glass?

11. The works of a watch are held 1.5 inches from a jeweler's eye, lens of focal length 1.75 inches. How many times are the works magnified? Of what sort is the image?

INTERNATIONAL ASTRONOMICAL UNION.

A delegation of American astronomers, headed by Professor W. W. Campbell, Director of the Lick Observatory, is now in attendance at a meeting in Brussels called for the purpose of organizing the International Astronomical Union. This new organization will take over and carry forward the work hitherto in the hands of the committee of the Carte du Ciel, the International Union for Cooperation in Solar Research, The Central Bureau for the exchange of Meteorological Telegrams, and other astronomical organizations. Sections of this Union will be organized in the several countries. The American section includes one or more representatives from each of the following organizations: The National Academy of Science, the American Astronomical Society, the American Mathematical Society, the American Physical Society, the Naval Observatory, and the Coast and Geodetic Survey. The American delegates at a preliminary meeting held in Washington, June 23-24, took up tentative suggestions regarding the reform of the calendar, and other matters to be presented at the meeting in Brussels.—*Popular Astronomy.*

PROGRAM, GENERAL SCIENCE SECTION, CENTRAL SCIENCE AND MATHEMATICS TEACHERS' ASSOCIATION, CHICAGO ILL., NOVEMBER 28 AND 29, 1919.

FRIDAY, NOVEMBER 28, 1:00 P. M.

1. "Air Conditioning in Modern School Buildings"—Mr. S. R. Lewis, Consulting Engineer, Chicago, Ill.
2. "Use of Physical Equipment of the Home and School Building in Classroom Instruction"—Dean E. S. Keene, North Dakota Agricultural College.
3. "Possibilities of Home Work in General Science"—Prof. G. A. Bowden, University School, Cincinnati, Ohio.
4. "Some Tangible Results from a General Science Course"—Mr. George Mounce, LaSalle-Peru Twp. High School, LaSalle, Ill.
5. Report of Committee on Reorganization of Science in the High School—Secretary of the Section, Miss Ada L. Weckel.
6. Business; appointment of committees.

SATURDAY, NOVEMBER 29, 10:00 A. M.

1. Business; election of officers.
2. "The Role of Laboratory Work in General Science, Teacher Training It Involves"—Prof. Herbert Brownell, University of Nebraska, Lincoln, Neb.
3. "General Science in the High School of Tomorrow"—Mr. J. Calvin Hanna, State Supervisor of High Schools, Springfield, Ill.
4. "Relation of General Science to the Smith-Hughes Courses"—Prof. A. W. Nolen, University of Illinois, Urbana, Ill.
5. Discussion of reorganization of science courses and the work for next year

PROBLEM DEPARTMENT.

Conducted by J. O. Hassler,

Crane Technical High School and Junior College, Chicago.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and solve problems here proposed. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. If you have any suggestion to make, mail it to him. Address all communications to J. O. Hassler, 2337 W. 108th Place, Chicago.

Problem 621 (October) should read in the first equation $2xz$ instead of $2yz$.

SOLUTION OF PROBLEMS.

611. *Proposed by A. MacLeod, Aberdeen, Scotland.*

Solve $\quad 2b(\sqrt{x+a}-b)+2c(\sqrt{x-a}+c) = a.$

I. *Solution by R. M. Mathews, Duluth, Minn.*

The equation may be rewritten as

$2(b\sqrt{x+a}+c\sqrt{x-a}) = a+2(b^2-c^2).$

Square and simplify to

$8bc\sqrt{x^2-a^2} = a^2+4(b^2-c^2)^2-4x(b^2+c^2).$

When this equation has been squared it may be arranged as

$$16x^2(b^2-c^2)-8x(b^2+c^2)[a^2+4(b^2-c^2)^2]+a^4+16(b^2-c^2)^4+8a^2(b^2+c^2)^2+32a^2b^2c^2 = 0.$$

The discriminant of this equation is

$$64(b^2+c^2)^2[a^2+4(b^2-c^2)^2]^2-64(b^2-c^2)^2[a^4+8a^2(b^2+c^2)^2+16(b^2-c^2)^4+32a^2b^2c^2]$$

which may be simplified to

$$256b^2c^2[a^2-4(b^2-c^2)^2]^2.$$

Thus

$$x = \frac{(b^2+c^2)[a^2+4(b^2-c^2)^2]\pm 2bc[a^2-4(b^2-c^2)^2]}{4(b^2-c^2)^2}$$

$$= \frac{a^2+4(b+c)^4}{4(b+c)^2}\ b\neq c, \quad \text{or} \quad \frac{a^2+4(b-c)^4}{4(b-c)^2}\ b\neq -c.$$

The first of these roots checks in the original equation; the second does not unless the negative root of $x-a$ be used.

II. *Solution by C. E. Githens, Wheeling, W. Va.*

Let $\sqrt{x+a} = s$ and $\sqrt{x-a} = d$; then $a = (s^2-d^2)/2$.

Substituting in the original equation,

$2bs-2b^2+2cd+2c^2 = (s^2-d^2)/2.$

Clearing and extracting square root of each member,

$2(b+c) = s-d = \sqrt{x+a}-\sqrt{x-a},$

$\sqrt{x-a} = \sqrt{x+a}-2(b+c).$

Substituting again in the original equation,

$2b(\sqrt{x+a}-b)+2c[\sqrt{x+a}-2(b+c)+c] = a,$

$2(b+c)\sqrt{x+a} = 2b^2+4bc+2c^2+a,$

$$\sqrt{x+a} = \frac{a+2(b+c)^2}{2(b+c)},$$

$$x = \frac{a^2+4(b+c)^4}{4(b+c)^2}.$$

The following table of possible numerical values is interesting:

$(b+c)$	*either*	± 1,	± 1,	± 1,	± 1,	± 1,	etc.
	or	± 2,	± 3,	± 4,	± 5,	± 6,	etc.
a		4,	6,	8,	10,	12,	etc.
x		5,	10,	17,	26,	37,	etc.

The values of a and $(b+c)$ are arithmetical series and the values of x form a different series.

Also solved by A. Pelletier, M. G. Schucker, H. C. Whitaker, *and the* Proposer.

612. *Proposed by R. T. McGregor, Elk Grove, Cal.*

Find four mixed numbers in arithmetical progression whose common difference is two and the sum of whose squares is a square.

Solution by Herbert C. Whitaker, Philadelphia, Pa.

Let $x-3$, $x-1$, $x+1$ and $x+3$ be the numbers; then $4x^2+20 = $ a square $= k^2$; $x = 1/2\sqrt{k^2-20}$. If it is desired to have the four numbers to be rational, then $k^2-20 = (k-p)^2$ whence $x = (p^2-20)/4p$ and the four numbers are $(p^2-12p-20)/4p$, $(p^2-4p-20)/4p$, $(p^2+4p-20)/4p$, $(p^2+12p-20)/4p$ in which p may have any value whatever.

If $p = 10$, the four numbers are integers.

If $p > 6+\sqrt{56}$, the four numbers are positive.

If $p = 14$, the four numbers are $1/7$, $15/7$, $29/7$, $43/7$.

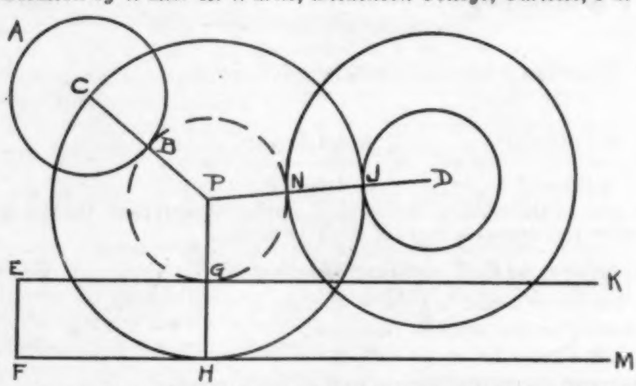
Also solved by Norman Anning, C. E. Githens, R. M. Mathews, A. Pelletier *and the* Proposer.

613. *Proposed by N. P. Pandya, Amreli, Kathiawad, India.*

Describe a circle to touch two given circles and a given straight line.

[We had hoped that some new methods of solution would be discovered by our contributors, but only the traditional solution was received.—*Editor.*]

I. *Solution by Walter R. Warne, Dickinson College, Carlisle, Pa.*



Given circles AB, ND and straight line EK. Let C and D be the centers of circles AB and ND, respectively, CB and DN their respective radii, and DN > CB.

With center D and radius DJ = DN − CB = NJ describe circle DJ. At E, and on the side of EK opposite C, erect EF (= CB) perpendicular to EK. Draw FM parallel to EK. Having first proved problem 603b, we can now draw a circle with center P, say, touching DJ, straight line FM in H, say, and passing through C.

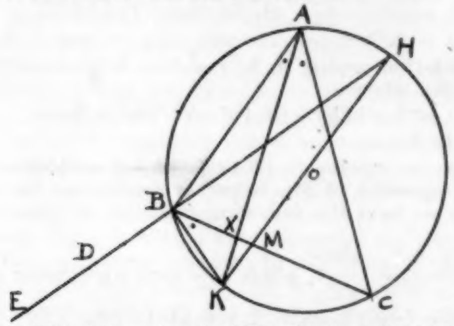
PH is perpendicular to FM. PH is perpendicular to EK at G, say. PH = PC = PJ. Since EF = GH = CB = NJ, PG = PB = PN. Whence circle BNG is the circle required.

Also solved by A. Pelletier *and* M. G. Schucker.

614. *Proposed by Daniel Kreth, Wellman, Iowa.*

Haying given the base, the line bisecting the vertical angle, and the diameter of the circumscribing circle, to construct the triangle and determine its sides.

Solution by A. MacLeod, Aberdeen, Scotland.



Given the base BC of the triangle ABC, the length of AX which bisects the angle A (l), and the radius of the circumcircle (R).

Construction:

Draw a circle, center O, radius R, and draw the chord BC and the diameter HK perpendicular to BC. Produce HB to D making BD $= l/2$ and to E making DE = DK. Draw the chord KA = BE. A is the vertex of the required triangle.

Proof:

Since $\angle KBX = \angle A/2$, $KA \cdot KX = \overline{KB}^2 = \overline{KD}^2 - \overline{BD}^2 = \overline{DE}^2 - \overline{BD}^2 = BE(DE-BD)$.

$\therefore KX = DE - BD$. *i. e.* $KA - AX = BE - 2BD$.

$\therefore AX = 2BD = l$.

To determine the sides.

I. Find AK from the equation $AK(AK-l) = \overline{KB}^2 = 4R^2\sin^2 A/2$. Then from $\triangle KBH$, $\cos(B-C)/2 = AK/2R$ which gives $(B-C)/2$, and since $(B+C)/2 = 90° - A/2$, B and C are found. Finally, $b = 2R\sin B$, $c = 2R\sin C$.

II. Without using tables. Let HK cut BC in M.

$\sin A = a/2R$, $\cos A = \sqrt{1-a^2/4R^2}$, $OM = R\cos A$:

$\overline{BK}^2 = KM \cdot KH = 2R^2(1-\cos A)$, $\overline{BH}^2 = 2R^2(1+\cos A)$

$AK(AK-l) = 2R^2(1-\cos A)$ and $\overline{AH}^2 = 4R^2 - \overline{AK}^2$. In quadrilateral ABKH, $AB \cdot HK + BK \cdot AH = HB \cdot AK$ which gives AB.

Also solved by A. Pelletier *and* W. R. Warne.

615. *Proposed by Walter R. Warne, Carlisle, Pa.*

Eliminate θ and φ from

$c\sin\theta = a\sin(\theta+\varphi)$, (1)

$a\sin\varphi = b\sin\theta$, (2)

$\cos\theta - \cos\varphi = 2m$. (3)

I. *Solution by R. Sciobereti, Los Angeles, Cal., and The Proposer.*

From (2)

$$\sin\theta = a\sin\varphi/b$$

Substituting in (1), and expanding,

$$ca\sin\varphi/b = a(a\sin\varphi\cos\varphi/b + \sin\varphi\cos\theta)$$

whence

$$c = a\cos\varphi + b\cos\theta.$$

Solving this with (3),

$$\cos\theta = (c+2am)/(a+b),\ \cos\varphi = (c-2bm)/(a+b).$$

From (2)

$$b^2(1-\cos^2\theta) = a^2(1-\cos^2\varphi)$$

whence

$$(a-b)[c^2-(a+b)^2] = 4abcm.$$

II. *Solution by M. G. Schucker, Pittsburgh, Pa.*

From the relations (1) and (2) it is obvious that the relations expressed may be those of a triangle of sides a, b and c and θ and φ are the angles opposite a and b, respectively. Hence from (1) and (2)

$$\cos\theta = (b^2+c^2-a^2)/2bc \text{ and } \cos\varphi = (a^2+c^2-b^2)/2ac.$$

The complete relation among a, b, c and m is therefore expressed by substituting in (3), whence

$$(b^2+c^2-a^2)/2bc-(a^2+c^2-b^2)/2ac = 2m.$$

Remarks by the Editor.

The result may be written $(a+b+c)(a+b-c) = 4abcm/(b-a)$ which in view of the suggestion of Mr. Schucker concerning the triangle, leads one to note that we have the following relations between the sides of a triangle.

$$s(s-c) = \frac{abcm}{b-a}, \text{ where } s = \text{semi-perimeter and}$$

$$m = (\cos A-\cos B)/2, \text{ provided } a \neq b.$$

Also solved by E. Kesner, R. M. Mathews, A. Macleod, A. Pelletier, *and* H. C. Whitaker.

See error correction for 621 on page 755.

PROBLEMS FOR SOLUTION.

626. *Proposed by Walter McNelis, Philadelphia.*

Prove the following equations are consistent and solve.

$$\begin{aligned}
50e_1+250e_2+100e_4 &= 6\\
265e_3+200e_5-250e_2 &= 0\\
110e_5-100e_4-200e_6 &= 0\\
50e_1+265e_3+110e_5 &= 6\\
e_1-\quad e_2-\quad e_5 &= 0\\
-e_1+\quad e_4+\quad e_5 &= 0\\
-e_4+\quad e_6+\quad e_2 &= 0\\
e_3-\quad e_5-\quad e_6 &= 0
\end{aligned}$$

626. *Proposed by Daniel Kreth, Wellman, Iowa.*

If x be any prime number except 2, the integral part of $(1+\sqrt{2})x$, diminished by 2, is divisible by $4x$.

628. *Proposed by H. C. Peterson, Crane Technical High School and Junior College, Chicago.*

Suppose a rectangular sheet of paper has vertices indicated by A, B, C, D, consecutively. Fold corner A over along line BX, X any point in AD; fold again so that XD lies in the direction XB making crease XY (Y in BC or CD); open and fold again so that DC lies in line XB making crease EF. Prove that XY and EF make an angle of 45° with each other.

629. *Proposed by Walter R. Warne, Carlisle, Pa.*

Give a solution by means of elementary plane geometry of the following: If l, m, n be the perpendiculars from the center of the circumscribed circle on the sides of a triangle, show that

$$4(a/l+b/m+c/n) = abc/lmn.$$

630. *Proposed by Herbert C. Whitaker, Philadelphia.*

A ship at sea sights two lighthouses, one due north, the other due east. After sailing N.35°E., 3 miles, the ship is equally distant from the two lights; and sailing on the same course one mile further, the ship is found to be on the same straight line with the lights. What is the distance between the two lighthouses?

AMERICA'S OPPORTUNITY IN CHEMISTRY.

By L. J. Henderson,
Chemistry Department, Harvard University.

Germany had the leadership in chemistry and we have got to take it away from that country, and we can only do that by developing along lines that have been developed only in Germany on a large scale. We must have something like the German relationship—I don't mean in moral aspects but material aspects—between the universities and the industries, and chemistry in particular is a subject where it is not enough to be a trained and experienced applier of science. The country, if it is to be safe industrially and if it is to have the leadership intellectually in chemistry, has got to have a great number of men who are primarily scientific chemists, not primarily engineers, but men who are able to apply all the great principles of science directly to the problem of the moment, just as has been done by many during the war. More and more as time goes on, that has come to be necessary. Now every university in America is capable of producing men of that kind, but there are very few that have anything more than a machine standard in their college. You have got to produce a man who is scientifically and intellectually an individualist, who has independent judgment and power. Such a man can be the product only of an institution or of a group of men who can give him something more than most colleges give, unless he is big enough to make himself. Harvard, we may justly claim, is one of the few institutions that has the capacity to produce such men. The country needs such men.

LESSONS IN DAIRYING FOR RURAL SCHOOLS.

To add impetus to the teaching of dairying in elementary rural schools the United States Department of Agriculture has just published Bulletin 763, which contains 12 lessons on the subject. With each lesson are given helpful directions for home projects that may be worked out with profit to every community and with real educational value to pupils. Practically all the subject material for class use and instructions for home projects can be found in available bulletins either free or at small cost, but teachers and pupils are advised to use additional sources of information, such as the printed matter from dairy cattle breeders' associations, books on dairying, and farm and dairy journals.

According to the bulletin teachers of agriculture are agreed that instruction on that subject should follow certain definite lines—it should be seasonal, be local in its interests, meet the needs of the pupils, and be practical. The home-project plan affords the best means of meeting these conditions, especially the practical side, for by it the pupil works out for himself the principles and theories taught in the classroom.

To Make Work Effective.

The term "home project," applied to instruction in elementary and supplementary agriculture, includes as requisites a plan for home work and related instruction in agriculture at the school. It should be a problem new to the pupil; the parent and pupil should agree with the teacher on the plan; some competent person must supervise the home work; detailed records of time, method, cost and income must be honestly kept; and a written report based on the record should be submitted to the teacher.

One of the means by which teachers may learn the dairy interests of the district is a dairy survey. The pupils may assist in obtaining this information, but first-hand knowledge obtained by the teacher will be valuable. This survey should tell the kind of farm (crop or stock), purpose of dairy

cows (commercial or home use), breeds of cattle, feeds raised, feeds purchased, milk records kept, how milk is tested, how milk is disposed of, and dairy conveniences. Information should be tabulated as it is collected. In addition the teacher with the pupils' help should provide charts showing the points of a typical dairy cow, samples of dairy records showing how they should be kept, directions showing the food value of milk, and drawings showing a section of the model dairy farm, milk house, etc.

LESSONS IN BULLETIN.

The bulletin contains lessons, giving sources of material, on the following subjects: Producing clean milk, care of milk and cream, weighing milk, testing milk, keeping of records and marketing dairy products, profit and loss (good and poor cows), judging and purchasing stock, care in handling of the dairy cow and barn, butter manufacture, food value of milk and its use in the home, making cottage cheese, cooking with milk products, and the use of milk as a supplement to other foods.

NEW OUTLINE MAP OF THE UNITED STATES ON THE LAMBERT PROJECTION.

The U. S. Coast and Geodetic Survey, Department of Commerce, reports the completion of the new outline map of the United States on the Lambert Conformal Conic Projection, scale 1-5,000,000, dimensions, 25x39 in., price, 25 cents.

This map is intended merely as a base to which may be added any kind of special information desired. The shoreline is compiled from the most recent Coast and Geodetic Survey charts. State names and boundaries, principal rivers, capitals and largest cities in the different states, are the only information otherwise embodied.

The map is of special interest from the fact that it is based on the same system of projection as that which was employed by the armies of the allied forces in the military operations in France. To meet those requirements and at the request of the Army, special publications were prepared by the Coast and Geodetic Survey.

Many methods of projection have been designed to solve the difficult problem of representing a spherical surface on a plane. As different projections have unquestionable merits as well as equally serious defects, any region to be mapped should be made the subject of special study and that system of projection adopted which will give the best results for the area under consideration.

The Mercator Projection, almost universally used for nautical charts, is responsible for many false impressions of the relative size of countries differing in latitude. The polyconic projection, widely used and well adapted for both topographic and hydrographic surveys, when used for the whole of the United States in one map, has the serious defect of unduly exaggerating the areas on its eastern and western limits.

Along the Pacific Coast and in Maine the error in scale is as much as 6½ per cent, while at New York it reaches 4½ per cent.

The value of the new outline map on the Lambert projection can best be realized when it is stated that throughout the larger and most important part of the United States, that is, between latitudes 30½° and 49°, the maximum scale error is only one-half of one per cent. This amount of scale error of one-half of one per cent is frequently less than the distortion due to the method of printing and to changes from the humidity of the air. Only in southernmost Florida and Texas does this projection attain its maximum error of 2½ per cent.

The Lambert projection is well adapted to large areas of predominating

east and west dimensions as the United States where the distance across from east to west is 1 $^4/_5$ times that of the distance north and south.

The strength of the polyconic projection on the other hand is along its central meridian. The merits and defects of the two systems of projection may be stated in a general way as being at right angles to each other.

Special features of the Lambert projection that are not found in the Polyconic may be stated briefly as follows:

1. The Lambert projection is conformal, that is, all angles between intersecting lines or curves are preserved; and for any given point (or restricted locality) the ratio of the length of a linear element on the earth's surface to the length of the corresponding map element is constant for all azimuths or directions in which the element may be taken.

2. The meridians are straight lines and the parallels are concentric circles.

3. It has two axes of strength instead of one, the standard parallels of the map of the United States being latitudes 33° and 45°, and upon these parallels the scale is absolutely true. The scale for any other part of the map, or for any parallel, can be obtained from Special Publication No. 52, page 36, U. S. Coast and Geodetic Survey. By means of these tables the very small scale errors which exist in this projection can be entirely eliminated.

THE DANGERS OF THE POSTAL ZONE LAW.

By Senator Arthur Capper,

of Kansas.

There is no subject of greater importance to the public than that involved in the postal principles on which is based our postal legislation. The present postal zone law needs careful consideration, and every citizen and home throughout this nation should earnestly endeavor to understand the important factors involved.

For there is no function of government that reaches every citizen and every home to the extent of our United States postal service. For over seventy years the history of our postal legislation shows that our country has not legislated for postal service on the basis of cost, because the postal service is of such universal benefit, is such an instrument of information and education and unification, that to restrict it in any way is to hurt the country that we as thinking citizens wish to serve. So clearly and firmly has this American postal principle been held, that postage cost must not determine the postage rate, that our post-office has delivered letters and publications to Yankee whaling ships at Point Barrow in the Arctic Circle for two cents that cost over $5.60 to deliver. I would ask any thinking citizen if it is not just as important that a Yankee skipper home from a whaling cruise shall be able to understand and vote intelligently upon the great public questions of the day as it is for the citizen who has stayed at home? This principle is sound. Shall not California, Kansas and Maine have equal postage on all information as an American right?

Our rural free delivery system—the most expensive and least revenue-producing branch of the post-office—costs 1½ cents per piece of mail matter, and this 1½ cents is over and above the cost of collecting, sorting, handling, transporting and rehandling until it gets into the rural free delivery carrier's wagon. This has all been done upon the American postal theory that the post-office function was a service to the American people and that the cheapness of postage was a benefit to the American home.

It has been alleged—and maybe some have fallen victim to its un-American and illogical absurdity—that cheap postage on magazines and newspapers is a subsidy to the publishers. It is not a subsidy to the publishers. It is, if you want to use the term "subsidy," a subsidy to American readers. You can determine this for yourself. Who receives the benefit or subsidy when the Yankee skipper of a whaling ship off Point Barrow, in the Arctic Circle, receives news from home which costs $5.60 to deliver? Is that a subsidy to his home newspaper, his periodical or magazine, or is the benefit of that to the ship captain himself and his citizenship and our united and national standards of intelligence?

You will instantly recognize that it is this ship captain receiver of costly postal service who is benefited, and your common sense will instantly prove to you that in every case of cheap postage the primary and entire benefit is to the receiver. Would you have Kansas pay higher postage than New York merely because any information happened to be printed in New York? Why handicap the postal service of Kansas by a higher and discriminatory postage rate? I come from Kansas, but the discrimination is similarly true of every other State.

Cheap postage on periodicals and newspapers has made the American nation a nation of readers beyond any nation in the world. If there is any thought in your mind that *this* is *not* a national benefit, I ask you to compare in your mind this great country with its splendid and homogeneous American idealism, its singleness of purpose and the universality of its achievements with those nations in the world in which there is but little magazine reading.

Now as a practical proposition. You know the economic law that all costs must ultimately be paid by the final consumer, i. e., in this case the reader. To raise the postage on publications means that the publishers, as business men, must add this charge to the price of their periodicals—and thus lessen reading. Is this a good thing? And again I ask every reader to consider those nations in the world which have never encouraged widespread reading nor the widespread distribution of periodicals and newspapers, and to answer that question. For it is one which I and other legislators in Congress have to face and with which we must deal.

This country had a postal zone system at one time, applying to letters and newspapers and periodicals. The abolition of the zone system was made complete by President Lincoln in 1863 and the zone system was abolished not only on periodicals and newspapers, but also on letters, because it was regarded as an unsound postal policy and un-American that a citizen or home should have to pay more postage simply by an accidentally greater distance from the point of mailing. The postal service is an American service from all Americans to all Americans on a basis of equal postage and equal service. I ask every reader to consider for himself if this is not sound Americanism.

Now on the practical side I wish to point out that the country newspapers have circulation in their county of publication without any postage charge whatsoever and this can only be justified and continued on our American theory that the postal function is an equal service to all American homes.

It would be obviously unfair for those supporting the postal theory that the cost must determine the rate of postage to ask that a letter costing 1½ cents for delivery alone on rural routes should be sent for one cent. I do not have to be convinced that we should have one cent letter postage. I am for cheap postage as a great American social service. I believe that every right-thinking American is for cheap and equal postage. But there is no logical reason for believing that the rate on one class of postal

matter must be determined by the rate on another class of postal matter. The figures of postal cost upon which this unsound and un-American postal cost theory is demanded were compiled in 1907 and upon being investigated by the United States Postal Commission headed by Hon. Charles E. Hughes, these figures were discarded as utterly unreliable in determining the cost of handling newspapers and periodicals. Yet it is upon these discarded cost figures that such unsound arguments are based.

If we must abolish postal service—or increase postage rates to a prohibitive basis—on the theory that cost of service shall determine the postage rates, we should have to abandon many of the most important of our postal functions, the rural free delivery being the most conspicuous example and one which I believe should be kept up no matter what its cost, as it is the most important postal service in the entire department. It pays too high a return—as does every other postal service—in improved and elevated citizenship.

I earnestly hope that every reader will give this postal zone matter and its revival of unsound postal theories that have been discredited for over two generations very serious thought.

THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.

The National Committee on Mathematical Requirements was organized in the late summer of 1916 for the purpose of giving national expression to the movement for reform in the teaching of mathematics which had gained considerable headway in various parts of the country.

The membership of the Committee at present is as follows:

Representing the colleges:

A. R. Crathorne, University of Illinois.

C. N. Moore, University of Cincinnati.

E. H. Moore, University of Chicago.

D. E. Smith, Columbia University.

H. W. Tyler, Massachusetts Institute of Technology.

J. W. Young, Dartmouth College (Chairman).

Representing the secondary schools:

Vevia Blair, Horace Mann School, New York. (Representing the Association of Teachers of Mathematics in the Middle States and Maryland).

W. F. Downey, English High School, Boston. (Representing the Association of Teachers of Mathematics in New England).

J. A. Foberg, Crane Technical High School, Chicago (Vice Chairman). (Representing the Central Association of Science and Mathematics Teachers).

A. C. Olney, Commissioner of Secondary Education, Sacramento, California.

Raleigh Schorling, The Lincoln School, New York.

P. H. Underwood, Ball High School, Galveston, Texas.

Eula Weeks, Cleveland High School, St. Louis, Mo.

Last May, the Committee was fortunate in securing an appropriation of $16,000 from the General Education Board, which has made it possible greatly to extend its work. This work is being planned on a large scale for the purpose of organizing a nation-wide discussion of the problems of reorganizing the courses in mathematics in secondary schools and colleges and of improving the teaching of mathematics.

J. W. Young and J. A. Foberg have been selected by the Committee to devote their whole time to this work during the coming year. To this end they have been granted leaves of absence by their respective institutions.

The following work is being undertaken immediately.

1. To make a careful study of all that has been said and done here and abroad in the way of improving the teaching of mathematics during recent years.

2. To prepare a bibliography of recent literature on the subject.

3. To make a collection of recent text books on secondary school and elementary college mathematics.

4. To prepare reports on various phases of the problem of reform. Eleven such reports are already under way and others are being projected.

5. To establish contact with existing organizations of teachers with the purpose of organizing a nation-wide study and discussion of the Committee's problem. The Committee hopes to induce such organizations to adopt this problem as their program for the year. It is ready to furnish material for programs and also to furnish speakers at meetings. The organizations in their turn are to furnish the Committee with the results of their discussions and any action taken. In this way it is hoped that the Committee can act as a clearing house for ideas and projects and can be of assistance in coordinating possible divergent views entertained by different organizations.

6. To promote the formation of new organizations of teachers where such organizations are needed and do not exist at the present time. These organizations may be sectional, covering a considerable area, or they may consist merely of local clubs which can meet at frequent intervals for the discussion and study of the problems of the Committee. It is hoped that such clubs can be organized in all the larger cities where they do not already exist.

7. To establish contact directly with individual teachers. The Committee feels that this is necessary in addition to their work through organizations in order to induce such individuals to become active and in order to make the work through organizations effective. Plans for establishing this contact with individuals on a large scale are under consideration, possibly through the publication of a Bulletin. These plans, however, are as yet in a tentative stage.

Organizations can be of assistance by sending to the Committee a statement of the name of the organization, its officers for the coming year, the time and place of its meetings and information regarding proposed programs. If any organization has within the last ten years issued any reports on topics connected with the work of the Committee, copies of such reports should, if available, be sent both to Mr. Young and Mr. Foberg. If this is impossible, a statement regarding the character and place of publication of any such reports would be welcome.

Individuals can be of assistance

1. By keeping the Committee informed of matters of interest that come to their notice;

2. By suggesting ways in which the Committee can be helpful;

3. By sending to the Committee in duplicate reprints of any articles they publish on subjects connected with the Committee's work;

4. By furthering the work of the Committee among their colleagues, organizing discussions, etc.

It is not too much to say that the existence of this Committee with its present resources gives the teachers of mathematics, both individually and through their organizations, a unique opportunity to do really constructive work of the highest importance in the direction of reform. They can surely be counted on to make the most of this opportunity.

BOOK REVIEWS.

Essentials of Arithmetic, by Samuel Hamilton, Ph. D., Superintendent of Schools, Allegheny County, Pa. First Book, Pages 8+368. 14×19 cm. *Second Book,* Pages 8+428. 14×19 cm. American Book Company, New York.

The *First Book* covers the work usually taught in the second, third, fourth, and fifth years, each chapter representing a half year's work. The motivation of the drill work, especially in the earlier grades, is secured by means of interesting number games. The problems at first deal with the pupil's life at home, at school, on the street, and in the playground, and gradually reach out to include his contact with social and industrial life.

The *Second Book* covers the work usually taught in the sixth, seventh, and eighth years, each chapter representing a half year's work. By the end of the sixth year this book aims to give the pupils a knowledge of the arithmetical principles that are essential for success in the ordinary affairs of life. The work of the seventh and eighth years makes wider applications of those principles in problems dealing with taxation, insurance, investments, and other business and social enterprises. In both books it has been the aim to make the problems real by making them true to life. The pupils' self-activity is utilized in constructive work and in the framing of original problems. H. E. C.

Junior High School Mathematics, Second Book, by E. H. Taylor, Ph. D., Instructor in Mathematics, and Fiske Allen, A. M., Principal of the Training School, Eastern Illinois State Normal School. Pages ix+231 13×19 cm. 1919. Henry Holt and Company, New York.

The *Second Book* continues the work in arithmetic, algebra, and geometry begun in the *First Book*. The first chapters give sufficient practice in the fundamental processes with algebraic symbols to prepare the

pupil to use formulas, equations, and graphs as tools for solving problems. The processes of arithmetic, algebra, and geometry are unified in the study of ratio and proportion, similar figures, and the mensuration of surfaces and solids. While the problem material here is taken largely from geometry the explanations and methods of solution are simplified by the use of the equation and the algebraic notation. The more difficult parts of business arithmetic are given at the end of the eighth year, and an effort has been made to use as much as possible of the pupil's knowledge of algebra. The choice of material has been determined by the demands of everyday life of the average citizen. The last few sets of problems are involved in projects planned to follow as closely as possible a series of activities requiring much computation for a genuine purpose. H. E. C.

Teachers' Manual, First Course in Algebra, by Walter B. Ford, Professor of Mathematics in the University of Michigan, and Charles Ammerman, The William McKinley High School, St. Louis, Mo. Pages 341. 13×19 cm. 1919. The Macmillan Company, New York.

In this manual not only are the solutions of all problems and exercises given, usually in full, but suggestions of a supplementary nature are also given so that the teacher may understand the authors' own point of view upon the various topics treated. H. E. C.

Junior High School Mathematics, Third Course, by William L. Vosburgh, Head of Department of Mathematics, The Boston Normal School, and Frederick W. Gentleman, Late Junior Master, Department of Mathematics, The Mechanic Arts High School, Boston. Pages ix+295. 13×19 cm. 1919. The Macmillan Company, New York.

This book has been planned to meet the needs of the first year mathematics in the ordinary high school, as well as to serve as a third course in junior high school mathematics. In fact it furnishes better work in mathematics than is usually given in the ordinary high school since some relatively useless topics in algebra are omitted from the algebraic portions, geometric matter of admitted value has been included, and the two subjects are presented in a way to give the pupil ability to use his knowledge. The problems have been selected with the idea that by means of them the pupil shall develop the ability to apply general principles to new situations, shall become proficient in the use of a variety of mathematical tools, and shall acquire an appreciation of the quantitative phases of his environment. H. E. C.

Applied Calculus, by Robert G. Thomas, Professor of Mathematics and Engineering at the Citadel, The Military College of South Carolina. Pages xvi+490. 13×19 cm. Flexible fabrikoid covers. Price, $3.00. 1919. D. Van Nostrand Company, New York.

The author, who was a practicing engineer, has for many years been teaching to college students the calculus and its applications and has prepared this text primarily for technical students. The applications and concrete illustrations have been chosen to illustrate and enforce the essential principles; while they have been taken from geometry, mechanics, general physics, and astronomy no special knowledge of these subjects is required on the part of the student.

No rigid distinction has been made between theory and practice but they have been treated alternately and the applications are interspersed throughout the text. It has been the purpose of the author to produce a text in line with modern ideas and to meet the very recent demand for the early presentation of mechanical principles and their application in engineering practice to technical students. The study of this book should

develop in the student the ability to reduce engineering facts to mathematical form and to get fruitful results through the union of theory and practice. H. E. C.

Complete School Algebra, Revised Edition, by Herbert E. Hawkes, Ph. D., Professor of Mathematics in Columbia University, William A. Luby, A. B., Head of Department of Mathematics, Kansas City Polytechnic Institute, and Frank C. Touton, formerly Principal of Central High School, St. Joseph, Mo. Pages ix+507. 13×19 cm. Price, $1.40. 1919. Ginn and Company, Boston.

In this revision material for which there is no strong demand from teachers has been omitted and the entire work has been rewritten in the interest of greater simplicity and directness of appeal. The lists of exercises and problems are for the most part new and contain a larger proportion of easy exercises with simple results than in the first edition. In the character and position of the examples and hints the aim has been to help the pupil at the exact point where he needs it. The definitions and axioms have been expressed in the simplest language which is consistent with scientific accuracy. The large number of exercises and problems affords an opportunity for the teacher to make a selection that will satisfy the needs of various classes. H. E. C.

A History of Mathematics. Second edition, revised and enlarged. By Florian Cajori, Professor of the History of Mathematics in the University of California. Pages viii+514. 16×22 cm. 1919. The Macmillan Company, New York.

About two years before the publication of the present volume there appeared a "revised and enlarged edition" of Cajori's *History of Elementary Mathematics* which caused great disappointment in view of the fact that it embodied very few improvements on the earlier edition and the enlargement was insignificant. On the other hand, the present work should more than satisfy those who have been led to buy it on account of the fact that it was announced as "revised and enlarged." The pages are somewhat larger and more closely printed than those of the earlier edition, and the amount of reading matter is about twice as large even if the number of pages has been increased by only about 100.

The quarter of a century which intervened between the first appearance of Cajori's well and favorably known *History of Mathematics* and the publication of the present volume witnessed a marvelous increase in the facilities for securing an accurate knowledge of the history of our subject. Among these facilities the large mathematical encyclopedias in course of publication and the enlarged third series of the periodical known as *Bibliotheca Mathematica* deserve especial mention. During this quarter of a century Cajori has been unusually active in contributing to the advance of mathematical historical knowledge and hence the present work naturally bears many evidences of ripe historical scholarship.

The field covered is so enormous that it is impossible for one man, even after so many years of study, to give authoritative information on all points involved and hence Cajori had to depend largely on the results obtained by others. In the history of mathematics, above all other mathematical subjects, those who confine themselves to first-hand knowledge are forced to remain within very narrow limits. Fortunately, Cajori did not hesitate to furnish a large amount of second-hand knowledge by quoting freely the views of specialists on various subjects, and the wise selection of these views constitutes one of the chief merits of the book.

This edition should be of especial interest to the teachers of mathematics since it is the most comprehensive history of our subject in the

English language. It is written in an attractive style and includes many references to popular fables which are mentioned in many of the so-called historical notes of our textbooks. The rapid increase in such notes makes it more and more desirable for teachers to have a somewhat comprehensive knowledge of the history of our subject.

The scope of the present volume is about the same as that of the first edition except that much more space is here devoted to modern developments and especially to developments due to Americans. It is a pleasure to find here a record of the work that is now being done in our own midst, even if one finds also evidences of the fact that the Germans have been most active during recent decades in writing on mathematical history and have naturally emphasized their own contributions. An instance of this emphasis in the present volume is found on page 265 where it is stated that "it remained for K. F. Gauss to break down the last opposition to the imaginary." This statement would be more accurate if "in Germany" had been added to it.

Two very important mathematical movements which were started since the first edition was first published are the inauguration of the international mathematical congresses and the international commission on the teaching of mathematics. These movements are barely mentioned in the present edition. This is also true as regards the international efforts to classify the entire mathematical literature according to a universally accepted method. It should, however, be noted that Cajori had to limit himself in many ways, and, on the whole, he seems to have selected those developments which are likely to be most interesting to the majority of readers. As it is the book fills a decided need and should do much to further general mathematical knowledge.

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Two worth-while articles in "Science."

Our teachers of laboratory sciences who have not happened to run across the May 30 and the August 1 numbers of "Science" will do well to read and ponder upon the article by W. L. Estabrook of the Department of Chemistry of the College of the City of New York (p. 506) on, "The Freas System," May 30 number, and the article in the August 1 number, (p. 112) by Prof. A. A. Blanchard, of Massachusetts Institute of Technology on "Laboratory Instruction in Chemistry, its Aims and its Limitations." The second paper was written by way of comment on the first.

The nature of the first article can be gleaned from a quotation from it, namely, "The Object of the Freas system (as developed by Prof. Thomas B. Freas, of the Department of Chemistry of Columbia University) is fourfold. First to save the student's time by giving him all the chemicals and apparatus he needs at his bench, second to insure pure and clean chemicals, third to save chemicals by giving the student just the amount needed, and doing away with the wasteful and sloppy side shelf reagents bottle, and fourth, to relieve the instructor of those details, and thus to enable him to devote his entire time to teaching and research."

For those who have not access to "Science" or time to read the articles in full it may be said that the Freas system makes use of a "curator of supplies," one who "must have recognition, both in rank and salary to attract a man of character, ability and training in laboratory needs." Under the plan this curator may have several assistants if the laboratories are large. The curator and his assistants prepare "student kits" containing everything needed by one student during a term. These kits and the necessary apparatus are assigned to the students on "checking out day" by the curator's assistants who take a receipt for them. The student then locks up his kit in a locker provided with a padlock of the students'

own selection. Extra chemicals and apparatus are provided at the stock room where students may receive them by signing up for them. It would appear that the chief novelty in the system consists in applying to chemicals a system similar to the more usual systems of issuing apparatus.

Much unnecessary moving about the room is prevented by this plan and the four objects announced at the beginning of this account are quite successfully attained. The plan has been in operation at Columbia for some seven years.

Professor Blanchard's article begins with a brief review of the outlines of the history of the use of laboratory teaching in the study of science and notes briefly the advantages claimed for the laboratory method. The author then considers the Freas System as to whether it may not, while promoting efficiency of the factory type, fail to further the ends of laboratory teaching as well as some other system. Professor Blanchard says, "Modern American factory methods are marvelous when measured by the material output, but they do not rank so high when measured in terms of the welfare of the individual worker. . . . . . The writer does not see how a standardized routine of laboratory experiments can stimulate the intellectual development of a student any more than the intensive drive of production in a textile mill can stimulate the joy of life in the worker standing day after day in the same place over noisy looms." Professor Blanchard makes a strong point of the fact that the primary purpose of the laboratory was originally for research and while not claiming that the student laboratory is strictly such he nevertheless thinks that the more of the research spirit one can get into the work of the pupils the better.

If the work cannot be done in something of this spirit it might perhaps be better to have less laboratory work. If the editor may be permitted a word in regard to the issue he would suggest that all will concede that the main object of the Freas System, namely avoiding the use of the teacher-in-charge as a stockroom worker, is a most worthy one and that it should by all means be attained. A stockroom force of sufficient size and quality is an essential aid to this end. Much of the most commonly used apparatus should certainly be in the possession of the individual student and he should be held responsible for it. Other less used apparatus should be made as easily available as possible under the conditions. The editor cannot see, however, why it is necessary, given a well trained stockroom force, to serve up the breakfast food, so to speak, in individual containers. Such a proceeding is certainly to be favored in a fly infested dairy lunch room, but there is so much of valuable training of a sort most needed by a chemist, to be had where the student pours out, or weighs out his own portions of reagents or chemicals in the laboratory or makes his own dilutions and learns the necessary precautions, that the editor would be loath to surrender for him this privilege. If the waste and the "sloppy" conditions and the contamination of chemicals and reagents occur only at the beginning and become the means of a growth in grace, as it were, in things chemical, as they should do under a good teacher, they are not only to be expected but hoped for. The student's time could not be better spent.

The hard and fast limitations of a course in which it is the exception to have recourse to general supplies might well kill all initiative on the part of the student, who would thus allow himself to be satisfied with having done the required number of experiments and would not go aside to investigate co-lateral topics which, if he was really and naturally interested in them, might well yield him more profit than many required experiments.

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